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# Overhead labour costs in a neo-Kaleckian growth model with autonomous expenditures

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# Overhead labour costs in a neo-Kaleckian growth model with autonomous expenditures

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**Abstract:** One of the most notable features of income distribution is the widening wage differential among workers: there is a redistribution in favor of top management at the detriment of ordinary workers. The paper incorporates this distinction between overhead managerial labour and direct labour into a neo-Kaleckian growth model with target-return pricing, where an autonomously growing demand component ultimately determines the long-run path of an economy. Our aim is to explore the role of overhead labour costs in the coevolution of income distribution and economic growth. When overhead labour is taken into account, the share of profits becomes an increasing function of the rate of utilization of capacity. This implies that empirical research based on the post-Kaleckian specification of the investment function may fail to isolate the pure profitability effects and is likely to be biased in finding a profit-led regime.

Our model features convergence to a fully-adjusted position in the long run. This is achieved by simultaneous path-dependent adjustments, both in the normal rate of utilization of capacity and in the growth rate of sales expected by firms. We examine the parametric conditions under which the model achieves a wage-led growth regime, in the restricted sense that both the average rates of accumulation and utilization decrease during the transitional dynamics arising from an upward adjustment of the normal rate of profit. Moreover, it is shown that a more equitable wage distribution between the ‘working rich’ and the ‘working poor’ will strengthen the wage-led nature of the economy.

**Keywords:** *neo-Kaleckian, growth, overhead labour, autonomous expenditures, target-return pricing*

**JEL codes:** *E11, E37, O41*

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## 1. INTRODUCTION

The popular success of the book of Thomas Piketty (2014) has rekindled interest in the study of income inequality (and possibly wealth inequality) among all strands of economic thought. One of the key features of post-Keynesian economics is its concern with the effects of changes in functional income distribution on economic activity as well as the impact of the evolution of economic activity on income distribution. Over the last twenty years, an enormous literature has been developed by post-Keynesian authors on the relationship between the profit share, or the wage share, and economic activity or economic growth, both from a theoretical angle and through empirical studies (Lavoie and Stockhammer 2013). What the work of Piketty and his colleagues throughout the world has however also underlined is the importance of income inequality arising from an unequal income distribution within wage earners – a feature that had been left relatively unexplored by post-Keynesian authors.

Thus one of most notable feature of income distribution over the last 40 years is the widening wage differential among workers: there is a redistribution in favor of top management at the detriment of ordinary workers. How can this be taken into account within a macroeconomic model of growth and income distribution which, by tradition, only distinguished between wages and profits, or at best between wages, net profits and rentier income? In this paper we propose to subdivide wage-earners into two categories, overhead labour and direct labour. Overhead labour, or white-collar workers, will be associated with ‘rich’ households, and hence with persons holding managerial positions or what the Bureau of Labor Statistics (BLS) in the United States calls supervisory workers. By contrast, direct labour, or blue-collar workers, will be associated with ‘poor’ households, and hence as persons designated as non-supervisory workers.

Simon Mohun is one of the few scholars who has taken interest in this distinction at the empirical level. Table 1 below shows numbers for the year 2010 and is taken from data that he collected from the BLS and provided to one of the current authors, and which is at the origin of the study found in Mohun (2014). Table 1 shows that the distinction between non-supervisory and supervisory workers is not innocuous and has a substantial sociological and economic meaning. While supervisory workers represent less than 20 per cent of all workers (more exactly, 18 per cent), their average annual salary is over

four times that of non-supervisory workers, so that they gather nearly half of all the wages and salaries that were paid out in 2010. As one would expect from the work of Piketty, there has been a huge evolution through the years. Back in 1964, while supervisory workers also represented slightly less than 18 per cent of the workforce, their annual wage income was only 2.3 times that of non-supervisory workers, so that their share of total wages and salaries was less than one third. This is a further justification for devising macroeconomic models that can take some account of this evolution in the widening annual wage differential between workers.

*Table 1: Labour indices of non-supervisory and supervisory workers*

<b>Variable</b>	<b>Non-supervisory workers</b>	<b>Supervisory workers</b>
Number of individuals (thousands)	92 122	20 029
Total wages (billions of dollars per year)	3 765	3 288
Total hours worked (millions per year)	152 692	58 740
Hourly wage rate (dollars per hour)	24.66	55.97
Annual wage rate (dollars per year)	40 867	164 135
Hours worked per individual per year	1 657	2 932

Source: Computations based on data provided to one of authors by Simon Mohun

Our intent in this paper is to introduce the distinction between non-supervisory and supervisory workers into a neo-Kaleckian model of growth and distribution. While there are dozens of such models, only very few of them incorporate this distinction. The first authors to do so, but in static models, were Asimakopulos (1970) and Harris (1974). In growth models, the distinction was introduced by Rowthorn (1981), Kurz (1990),

Nichols and Norton (1991), Lavoie (1992, 1995, 1996a, 2009, 2014), and more recently Dutt (2012), Tavani and Vasudevan (2014), Nikiforos (2017) and Palley (2015, 2017).<sup>1</sup>

Relative to these previous works, the peculiarity of our paper is that the distinction between overhead and direct labour is set within a neo-Kaleckian growth model with autonomously-growing expenditures that do not by themselves create production capacity. There is now a general recognition that this may be an appropriate way to model basic stylized facts of modern economies with access to credit. Published papers developing this new approach, which is related to the so-called Sraffian supermultiplier of Freitas and Serrano (2015), can be found in Allain (2015, 2018), Lavoie (2016), Nah and Lavoie (2017) and Hein (2018), with a number of other papers extending this line of research in various directions, as in Dutt (2015), Lavoie (2017), Nah and Lavoie (2018a, 2018b) and Fazzari et al. (2018). Indeed, in a sense, this new variant in the way neo-Kaleckian models are being formalized is closer to the empirical work that has been pursued on the basis of the post-Kaleckian model of Bhaduri and Marglin (1990). Empirical works assess whether changes in income distribution have an effect on the *level* of economic activity while standard neo-Kaleckian growth models show that such changes affect the *growth rate* of economic activity. By contrast the new neo-Kaleckian growth models such as the one presented here show that changes in income distribution have an effect on the level of economic activity and not on its growth rate, except as a temporary phenomenon that occurs during the transition towards the new long-run equilibrium.

The next section presents the key equations of our model, notably how the amount of direct and overhead labour are determined, and it calculates the algebraic values of the various shares of income that go to profits, managers and direct labour when firms set prices on the basis of a target-return pricing formula. We shall see that the profit share and the share of income going to managers depend on the realized rate of capacity utilization. In the third section we analyze the determinants of the short-run equilibrium, having previously discussed the shape of the saving and investment functions. In particular we discuss the conditions under which an increase in the target rate of return or in the wage rate of managers relative to that of direct workers has a negative impact on

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<sup>1</sup> As far as we know, Rolim (2017) is the only empirical work on wage-led and profit-led demand regimes that splits the wage share into the shares going to supervisory and non-supervisory workers.

the rate of capacity utilization. In the fourth section, we shall study the long-run equilibrium of the model. At this stage we introduce two path-dependent adjustments, such that both the normal rate of capacity utilization and the growth rate of sales expected by firms react to realized values. We examine the parametric conditions under which the model achieves a wage-led growth regime, in the restricted sense that both the average rates of accumulation and of utilization decrease during the transitional dynamics arising from an upward adjustment of the target rate of return in the pricing equation. We also examine what happens in the long run when a less equitable wage distribution between the ‘working rich’ and the ‘working poor’ is being imposed to our model economy. In the fifth section, we report numerical simulation results for the calibrated model as an illustration. The last section will recap the main innovative features of the model and our main results.

## 2. THE MODEL ECONOMY

The model economy is a closed economy with no government, where households and monopolistically competitive firms interact. We borrow the basic assumptions from Lavoie (2009) for our artificial economy. Firms are owned and managed by a subset of households, who are the ‘working rich’ in the sense that they comprise shareholders and at the same time provide managerial, or supervisory, i.e., overhead labour services for the firms. On the other hand, the rest of households, who are the ‘working poor’, do not save and provide ordinary, i.e., direct labour services for the firms. The (nominal) wage rate is given by  $w$  for direct labour and  $\sigma w$  for overhead labour, where we assume  $\sigma > 1$ .

The number of direct workers,  $L_v$ , varies proportionally to the level of production,  $q$ , while that of overhead workers,  $L_f$ , hinges on productive capacity,  $q_{fc}$ . For some constants  $y_v$  and  $y_f$ , we have  $q = y_v L_v$  and  $L_f = q_{fc}/y_f$ . This implies that direct labour costs are variable and overhead labour costs are fixed with respect to  $q$ , once the stock of capital,  $K$ , is given. With  $f \equiv y_v/y_f$ , we can measure the size of  $L_f$  relative to  $L_v$  when the economy is operating at full capacity (when  $q = q_{fc}$ ). The productivity of direct labour is given by  $y_v$ .

National income is divided into profits and wages, and the latter is again divided into wages to overhead labour and wages to direct labour. Let  $\pi$  denote the net share of profits out of national income, while  $m$ , where  $m$  stands for ‘managers’, denotes the share of national income that goes to overhead labour. Hence,  $(\pi + m)$  amounts to the gross share of profits. From now on we will refer to  $\pi$  as simply the share of profits.

Let us assume that workers involved with direct labour, that is, poor households, save nothing and consume all their wages. In our model economy, total saving is then made up of three components: saving from firms, saving from rich households providing overhead labour, and a negative component, which is dissaving due to autonomous consumption by rich households. Let us further assume that firms retain a proportion  $s_f$  of their net profits, and that rich households save a constant proportion  $s_h$  of both components of their income, which includes overhead wages and the dividends that they receive from firms. These dividends are thus a proportion  $(1 - s_f)$  of net profits. We assume that  $0 < s_f < 1$  and  $0 < s_h < 1$ . With  $Z$  the real autonomous consumption expenditure, total saving can then be expressed as:

$$s_f \pi q + s_h [mq + (1 - s_f) \pi q] - Z \quad [1]$$

Dividing [1] by  $K$ , we have a saving function expressed in growth terms,  $g_s$ , such that

$$g_s = s_f \frac{\pi u}{\nu} + s_h \left[ \frac{mu}{\nu} + (1 - s_f) \frac{\pi u}{\nu} \right] - z \quad [2]$$

where  $u = q/q_{fc}$  is the rate of utilization of capacity,  $\nu = K/q_{fc}$  is the capital-to-capacity output ratio which is assumed to be constant, and where  $z = Z/K$  is the relative magnitude of autonomous expenditures compared to the volume of capital.

There is quite a bit of empirical evidence that firms use target-return pricing (Lee 1998). Under this pricing procedure, the price is set in such a way that a normal rate of profit  $r_n$  will be achieved when the economy is running at the normal rate of utilization of capacity, i.e., when  $u = u_n$ .<sup>2</sup> Lavoie (2014, p. 333) shows that under these conditions the

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<sup>2</sup> Target-return pricing is a specific version of normal-cost pricing, where the estimated unit cost is calculated at the standard rate of capacity utilization, which corresponds to normal output, also called nowadays *budgeted output* (Lee 2013).



price is given by:

$$p = \left[ \frac{u_n + \sigma f}{u_n - r_n v} \right] \frac{w}{y_v} \quad [3]$$

For [3] to be positive, it must be that

$$u_n > r_n v \quad [4]$$

Notice that  $(w/y_v)$  in the right-hand side of [3] is the nominal unit direct cost, and hence the term inside the bracket,  $(u_n + \sigma f)/(u_n - r_n v) > 1$ , is nothing but a mark-up over the unit direct cost. Obviously from [3], the inverse of this mark-up, which is in fact the real unit direct cost, amounts to the income share of direct labour,  $d$ , where  $d$  is for ‘direct labour’, since

$$d = \frac{\left(\frac{w}{p}\right) L_v}{q} = \frac{u_n - r_n v}{u_n + \sigma f} \quad [5]$$

Obviously, from [5], we can see that an increase in the target rate of return – the normal rate of profit  $r_n$  – will lead to a fall in the share of income going to direct labour. Similarly, when there is an increase in the proportion of supervisory workers (measured by  $f$  at full capacity) or in the wage premium going to managers (measured by  $\sigma$ ), the share of income going to direct labour gets reduced.

The income share that goes to overhead salaries,  $m$ , becomes

$$m = \frac{\left(\frac{\sigma w}{p}\right) L_f}{q} = \frac{\sigma f}{u} \left[ \frac{u_n - r_n v}{u_n + \sigma f} \right] \quad [6]$$

It follows that the share of profits is equal to

$$\pi = 1 - d - m = 1 - \left[ 1 + \frac{\sigma f}{u} \right] \left[ \frac{u_n - r_n v}{u_n + \sigma f} \right] \quad [7]$$

From [6] and [7], it is clear that the share of profits,  $\pi$ , is an increasing function of the rate of utilization of capacity,  $u$ , while the share of overhead salaries,  $m$ , decreases in  $u$ . With overhead labour present, an increase in the rate of utilization due to an increase in

effective demand brings about changes in the composition of labour, increasing the relative proportion of direct labour and decreasing that of overhead labour. This implies that the share of profits itself can be a poor indicator either of the expected profitability or of the relative bargaining position of capital in the class conflict.<sup>3</sup>

It was in this vein that Weisskopf (1979) suggested the concept of the ‘corrected share of profits’, which here we denote by  $\pi_c$ , and which we define as follows:

$$\pi_c = 1 - d - m \frac{u}{u_n} \quad [8]$$

Evidently,  $\pi_c$  corrects the actual share of overhead salaries by making it proportional to the actual utilization rate relative to its normal rate.<sup>4</sup> Hence, the uncorrected share of profits  $\pi$  and the corrected one  $\pi_c$  can be contrasted by comparing equations [7-1] and [8-1] below, obtained after some manipulations.

$$\pi = \frac{(\sigma f + r_n v)u - (u_n - r_n v)\sigma f}{u(u_n + \sigma f)} \quad [7-1]$$

$$\pi_c = \frac{r_n v}{u_n} \quad [8-1]$$

Notably, out of  $(d, m, \pi, \pi_c)$ , only  $d$  and  $\pi_c$  are independent of the rate of utilization while the others are not. In this respect, we argue that, when we consider the post-Kaleckian investment function of the Bhaduri-Marglin (1990) type, the true indicator of the expected profitability or of the relative power of capital in the class struggle should be  $\pi_c$  (or inversely  $d$ ), or rather  $r_n$  itself, instead of the traditional measure  $\pi$ . This is because, in the presence of overhead labour, the traditional share of profits  $\pi$  will not succeed in isolating the pure effects of profitability on accumulation, as has been repeatedly argued by Weisskopf (1979), Sherman and Evans (1984), and most recently by Lavoie (2017), to name a few.

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<sup>3</sup> As Palley (2017, p.50) puts it: ‘If the profit share is a positive function of capacity utilization, shifts of the IS schedule can make it look empirically as if the economy is profit-led when it is in fact wage-led’.

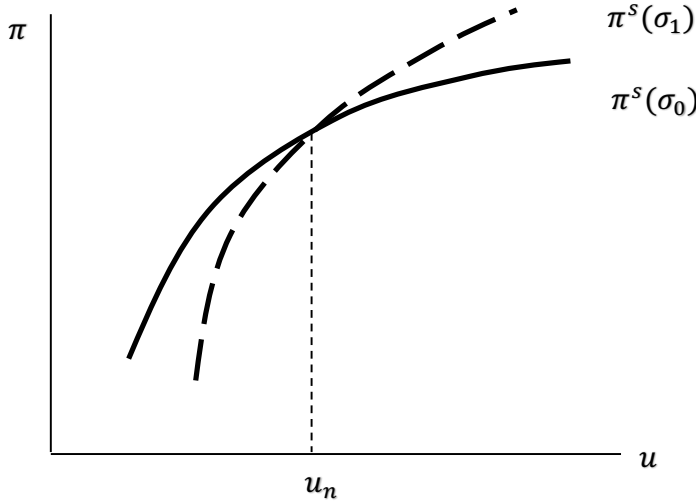
<sup>4</sup> Weisskopf (1979) defined the corrected profit share by multiplying the income share of overhead labour by the ratio of output relative to full-capacity output, but consistency with the target-return formula requires that it be multiplied by the ratio of output relative to normal output.

It is interesting to note that the corrected share of profits  $\pi_c$  is impervious to any change in the wage differential between supervisory and non-supervisory workers. By contrast, the share of profits  $\pi$  will move up or down as a consequence of an increase in the wage premium going to managers, measured by  $\sigma$ ,<sup>5</sup> depending on whether the actual rate of capacity utilization is higher or lower than the normal rate of utilization, as the following equation shows:

$$\frac{\partial \pi}{\partial \sigma} = \frac{f(u - u_n)(u_n - r_n v)}{u(u_n + \sigma f)^2} \quad [9]$$

Note that this partial derivative is valid for a given rate of utilization. This means that the profit-share curve seen from the supply side [7-1], drawn in  $(u, \pi)$  space, shifts down if  $u < u_n$  and shifts up if  $u > u_n$  as  $\sigma$  increases. This is illustrated in Figure 1.

Figure 1: Profit curves seen from the supply side with  $\sigma_0 < \sigma_1$



We may now deal with the last elements of the model. The saving function can be reformulated with an explicit consideration of our target-return pricing formula as can be found in Appendix 1. In this paper, we assume that the investment function takes the standard form of those neo-Kaleckian models that are concerned with a possible convergence towards the normal rate of capacity utilization.

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<sup>5</sup> This is also true of a change in the proportion of supervisory workers, measured by  $f$  at full capacity. However, we focus on  $\sigma$  because, as shown in the introduction, the variable  $f$  seems to have hardly changed through time.

$$g_i = \gamma + \gamma_u(u - u_n) \quad [10]$$

$g_i$  is the rate of accumulation of capital, i.e., investment divided by  $K$ . The first term on the right-hand side,  $\gamma$ , represents the secular expected growth rate of sales. The coefficient  $\gamma_u$  captures the sensitivity of induced investment to changes in effective demand, i.e., to changes in the rate of capacity utilization.

### 3. EQUILIBRIUM IN THE SHORT RUN

For analytical tractability, we introduce a distinction between the short and the long run into our model economy. We define the short run as the time period during which the capital stock,  $K$ , does not change, thus implying that there is no expansion of production capacity as of yet. It is also assumed that over the short run there is no change in the normal rate of capacity utilization,  $u_n$ , the expectation of the firms about future sales growth,  $\gamma$ , and the autonomous consumption by the working rich,  $Z$ . All these variables will be allowed to adjust in the long run.

The short-run equilibrium is achieved by adjustments in the rate of utilization of capacity so as to equate  $g_s$  to  $g_i$  (so that saving equals investment, or alternatively, so that aggregate supply equates aggregate demand). The equilibrium rate of utilization of capacity will be denoted by  $u^*$ , as determined in Appendix 1. For the short-run equilibrium to be stable, the following Keynesian stability condition must be met.

$$[s_f(1 - s_h) + s_h](\sigma f + r_n v) - \gamma_u v(u_n + \sigma f) \equiv A > 0 \quad [11]$$

We can also obtain the short-run equilibrium values of the share of profits,  $\pi^*$ , the rate of profit,  $r^*$ , and the rate of accumulation,  $g^*$ , as found in Appendix 1 .

A simple comparative statics exercise shows that both the short-run equilibrium values of the rate of capacity utilization and of the rate of accumulation are lower when there is a higher normal rate of profit  $r_n$ . This reveals the wage-led nature of demand and accumulation of the present model in the short run.

$$\frac{\partial u^*}{\partial r_n} = \frac{-\sigma f v \{s_f(1 - s_h) + u^*[s_f(1 - s_h) + s_h]\}}{A} < 0 \quad [12]$$

$$\frac{\partial g^*}{\partial r_n} = \gamma_u \frac{\partial u^*}{\partial r_n} < 0 \quad [13]$$

Furthermore, we can evaluate the short-run effects of widening the gap  $\sigma$  between the wage rate obtained by overhead labour and the wage rate of direct labour. We have

$$\frac{\partial u^*}{\partial \sigma} = \frac{f(u_n - r_n v) \{u_n s_f(1 - s_h) - u^*[s_f(1 - s_h) + s_h]\}}{A(u_n + \sigma f)}. \quad [14]$$

Three cases need to be considered:

(a)  $\partial u^*/\partial \sigma > 0$  and  $\partial g^*/\partial \sigma > 0$ , if

$$u^* < \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n, \quad [14-1]$$

(b)  $\partial u^*/\partial \sigma < 0$  and  $\partial g^*/\partial \sigma < 0$ , if

$$u^* > \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n, \quad [14-2]$$

And finally, (c)  $\partial u^*/\partial \sigma = 0$  and  $\partial g^*/\partial \sigma = 0$ , if

$$u^* = \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n. \quad [14-3]$$

A permanent increase in  $\sigma$  has a twofold effect. Obviously, it directly increases overhead expenses incurred by firms. This reallocates income from the firms to the working rich. With income going to firms, only the ratio  $(1 - s_f)(1 - s_h)$  gets to be spent on consumption by rich households; with income going directly to the households of the working rich, the ratio  $(1 - s_h)$  gets spent. The former is strictly less than the latter. Moreover, when the value of  $s_f$  is higher, the difference between the former and the latter gets bigger. Thus, an increase in  $\sigma$  may lead to an increase in consumption and in the rate of utilization as in the (a) case, and this positive redistribution effect is amplified with a

bigger  $s_f$ . We can see that, the higher is  $s_f$ , the more likely is the (a) case, since the term inside the squared bracket on the far right of (a) increases in  $s_f$ . Notice that  $u^*$  is more likely to be smaller once the term inside the bracket is big enough, and vice versa.

$$\frac{\partial}{\partial s_f} \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] > 0. \quad [15-1]$$

However, this is not the whole story. As is evident from [5], an increase in  $\sigma$  lowers the income share that goes to the households of the working poor, whose propensity to consume is assumed to be unitary. This is because of the target-return pricing strategy: as equation [3] shows, an increase in overhead costs will lead to an increase in the mark-up over direct unit labour costs and hence to a fall in the real wage of direct labour. A widening gap in wage rates shifts income away from the poor to the rich. Due to the difference in their respective propensities to save, this change yields a shrinking effect on the economy as in the (b) case. This is to be expected because if rich households receive higher wages but save most of it, the potential positive effect of this change will be that much smaller. Also, it is apparent that, as  $s_h$  gets smaller, the difference in the propensities to consume of the poor and the rich diminishes, and hence the negative effect caused by the redistribution towards the rich is dampened. This makes the (b) case less likely. It is consistent with the fact that the term in the squared bracket on the far right of (b) decreases in  $s_h$ .

$$\frac{\partial}{\partial s_h} \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] < 0. \quad [15-2]$$

These two effects are in conflict with each other. The former positive effect is due to the redistribution towards the working rich away from the retained earnings of the firms, and the latter negative effect is due to the redistribution towards the working rich away from the working poor. Unless the former dominates the latter, the less equitable structure of wages resulting from an increase in  $\sigma$  turns out to reduce effective demand in the short run, as in the (b) case. From the discussions above, we can conclude that the (b) case is more likely to occur than the (a) case when the marginal propensity to save of the working rich, which is  $s_h$ , is high and when the retention ratio of firms, which is  $s_f$ , is low. We can also conclude from the fact that the term inside the bracket of the right hand

side of either [14-1] or [14-2] is smaller than unity that in the neighbourhood of the normal rate of capacity utilization  $u_n$ , the effect on economic activity of an increase in wage dispersion will be negative.

Coming now to the effects on the profit share as a consequence of an increase in the wage premium obtained by overhead labour, i.e., of an increase in wage dispersion, we plug [6] into the saving function [2], thus obtaining a saving function with only the profit share and the rate of utilization as endogenous variables, and then equate this saving function to the investment function [10] to get the profit share as a function of the rate of utilization:

$$\pi = \frac{1}{s_f + s_h(1 - s_f)} \left[ v\gamma_u - s_h\sigma f \left( \frac{u_n - r_n v}{u_n + \sigma f} \right) \right] + \frac{v(\gamma - \gamma_u u_n + z)}{u[s_f + s_h(1 - s_f)]} \quad [7-2]$$

Equation [7-2] is the profit share seen from the demand side and can be contrasted with [7-1], which is seen from the opposite cost side. Evaluating the partial derivative with respect to  $\sigma$ , while holding the rate of utilization constant in [7-2], we have:

$$\frac{\partial \pi}{\partial \sigma} = - \frac{f s_h u_n (u_n - r_n v)}{[s_f + s_h(1 - s_f)](u_n + \sigma f)^2} < 0 \quad [16]$$

This implies that the profit-share curve seen from the demand side [7-2], drawn in  $(u, \pi)$  space, shifts down in a parallel fashion when  $\sigma$  increases, as is illustrated in either Figure 2A or Figure 2B, depending on whether the term  $(\gamma - \gamma_u u_n + z)$  in [7-2] is positive or negative.

Figure 2A: Profit curves seen from the demand side with  $\gamma - \gamma_u u_n + z > 0$  and  $\sigma_0 < \sigma_1$

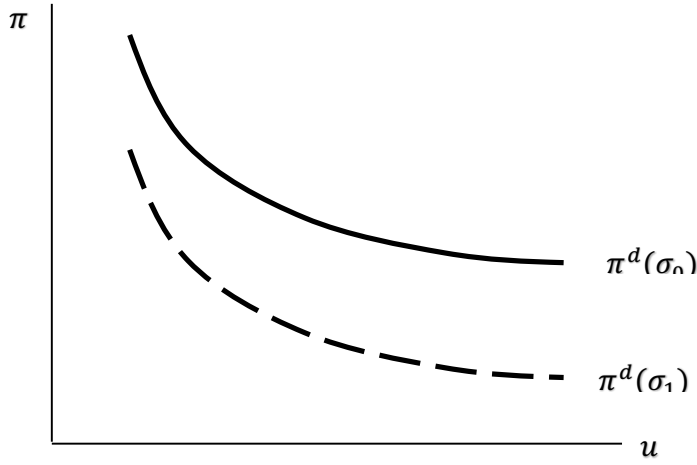


Figure 2B: Profit curves seen from the demand side with  $\gamma - \gamma_u u_n + z < 0$  and  $\sigma_0 < \sigma_1$

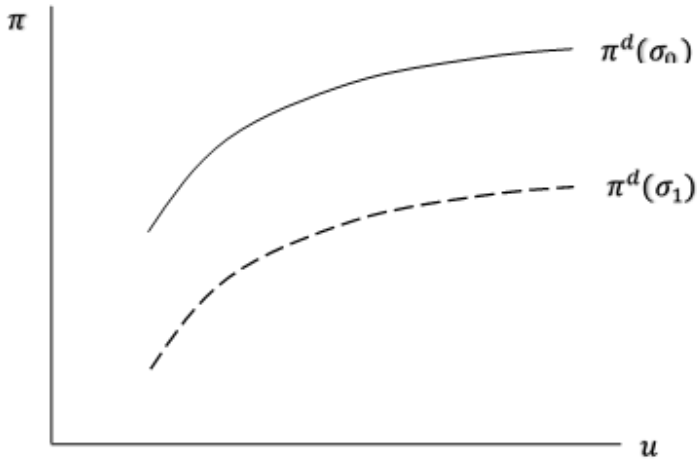


Table 2 summarizes the effects of an increase in  $\sigma$  (the wage premium of overhead labour) on the rate of utilization and on the share of profits when the combined effects on the demand and supply sides are taken into consideration. With regards to the impact of an increase in the wage differential on the short-run equilibrium share of profit, an analytical discussion is provided in Appendix 2. As to the impact of this increase on the rate of capacity utilization, the possible cases were explained above with the help of equations [14]. The detailed graphical illustrations corresponding to the various cases of Table 2 can be found in Figure 3A and Figure 3B.



Figure 3A (1): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z > 0$  and  $u > u_n$

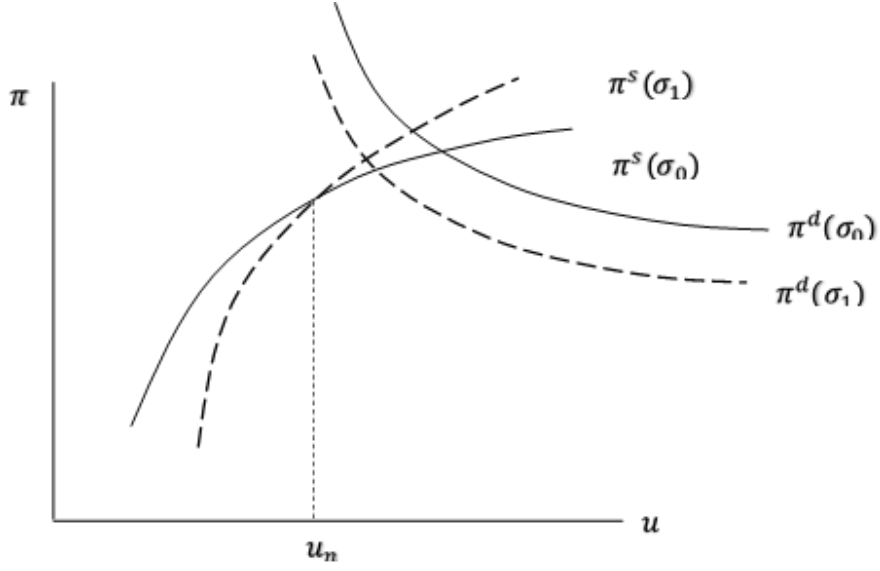


Figure 3A (2): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z > 0$  and  $\left[ \frac{s_f(1-s_h)}{s_f(1-s_h)+s_h} \right] u_n < u < u_n$

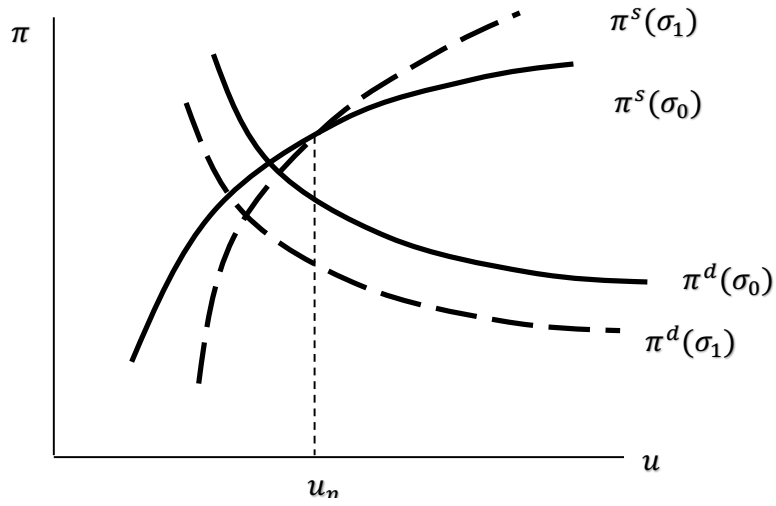


Figure 3A (3): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z > 0$  and  $u < \left[ \frac{s_f(1-s_h)}{s_f(1-s_h)+s_h} \right] u_n$

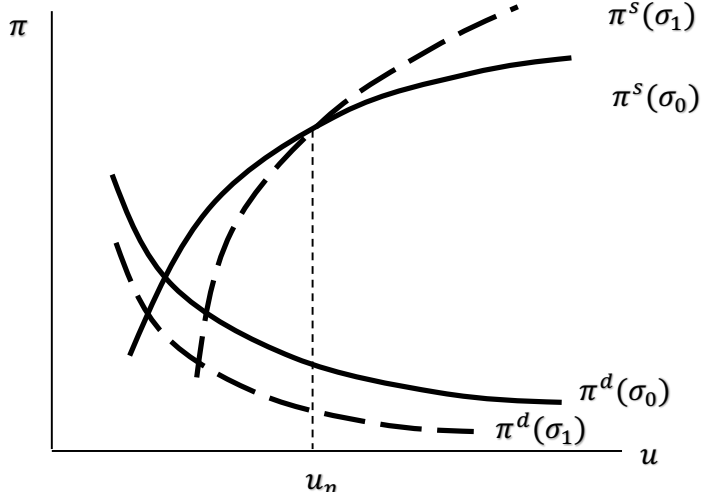


Figure 3B (1): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z < 0$  and  $u > u_n$

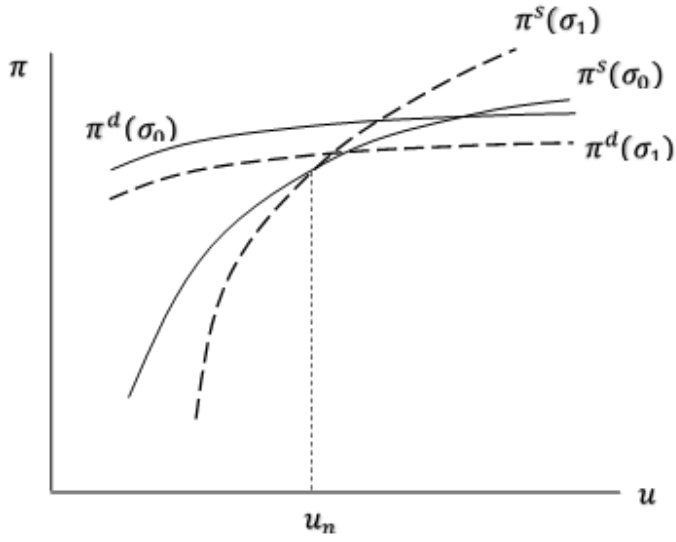


Figure 3B (2): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z < 0$  and  $\left[ \frac{s_f(1-s_h)}{s_f(1-s_h)+s_h} \right] u_n < u < u_n$

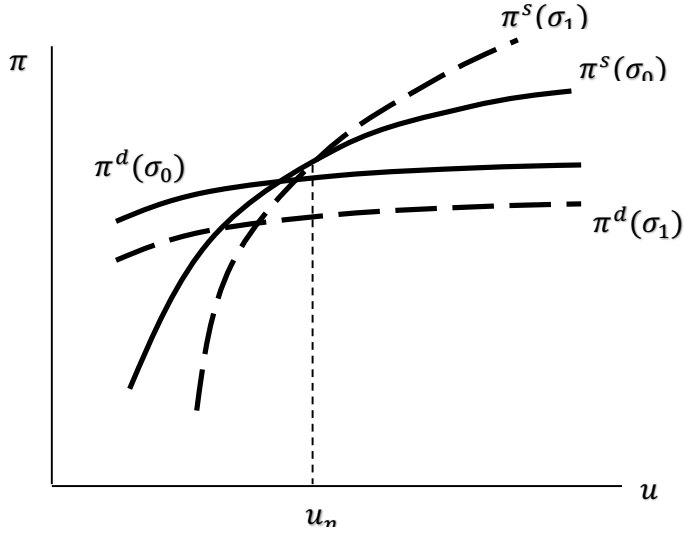


Figure 3B (3): The effects of an increase in  $\sigma$  for the case of  $\gamma - \gamma_u u_n + z < 0$  and  $u < \left[ \frac{s_f(1-s_h)}{s_f(1-s_h)+s_h} \right] u_n$

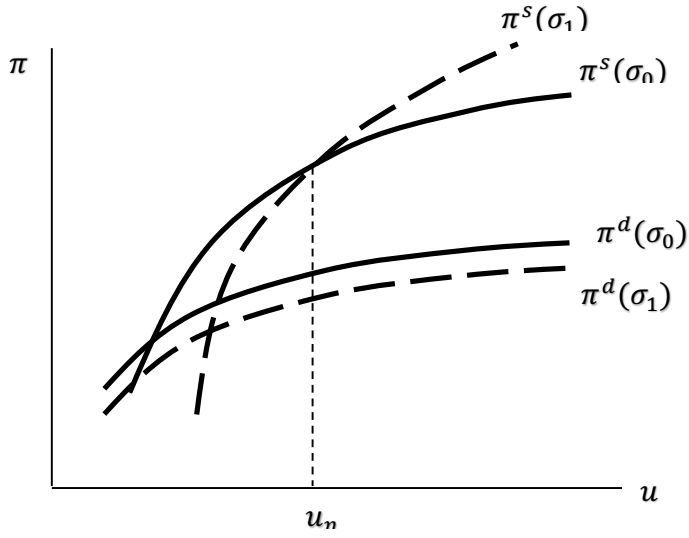


Table 2: The short-run effects of an increase in  $\sigma$ , the wage premium of overhead labour

	Utilization rate	$du$	$d\pi$	Figure
$\gamma - \gamma_u u_n + z > 0$	$u > u_n$	(-)	?	3A (1)
	$\left[ \frac{s_f(1-s_h)}{s_f(1-s_h) + s_h} \right] u_n < u < u_n$	(-)	(-)	3A (2)
	$u < \left[ \frac{s_f(1-s_h)}{s_f(1-s_h) + s_h} \right] u_n$	(+)	(-)	3A (3)
$\gamma - \gamma_u u_n + z < 0$	$u > u_n$	(-)	(-)	3B (1)
	$\left[ \frac{s_f(1-s_h)}{s_f(1-s_h) + s_h} \right] u_n < u < u_n$	(-)	(-)	3B (2)
	$u < \left[ \frac{s_f(1-s_h)}{s_f(1-s_h) + s_h} \right] u_n$	(+)	?	3B (3)

#### 4. EQUILIBRIUM IN THE LONG RUN

We now move on to a longer time horizon, and let autonomous consumption,  $Z$ , increase at the exogenously given rate of  $\bar{g}_z$ . In this paper, we define the ‘long run’ as the time period during which our artificial economy ultimately reaches a fully-adjusted position, in the sense that both the growth rate of sales expected by firms,  $\gamma$ , and the targeted rate of capacity utilization set by firms,  $u_n$ , are allowed to adjust, while the flow of autonomous consumption relative to the stock of capital, given by  $z$ , is allowed to move through time. These three variables change through time according to the laws of motion given by [17], [18] and [19], where “ $\wedge$ ” represents the rate of change, and where we assume that  $\phi_\gamma > 0$  and  $\phi_n > 0$ .

$$\hat{z} = \bar{g}_z - g^* \quad [17]$$

$$\hat{\gamma} = \phi_\gamma (g^* - \gamma) \quad [18]$$

$$\hat{u}_n = \phi_n (u^* - u_n) \quad [19]$$

Equation [17] arises straightforwardly from the definition of  $z = Z/K$ . Equation [18] implies that the sales expectations of firms react to the realized rate of growth. For instance, when the actual growth rate of the economy turns out to be unexpectedly high, the expected growth rate of sales will be modified upwards accordingly. This reaction function is often referred to as the Harrodian instability mechanism. Equation [19] provides a long-run endogenous adjustment in the normal rate of capacity utilization. The argument is that if the realized rates of utilization exceed persistently the rate construed as being normal, it seems quite possible that firms would change their view about what the normal rate of utilization ought to be, making it higher, both for their investment and pricing decisions. Symmetrically, if the actual rate of utilization is systematically below the normal rate, the convention about what the normal rate ought to be may be given a lower value.

A number of authors in the past have provided different arguments to the effect that the normal rate of capacity utilization is likely to be influenced by the values being taken by the actual rate of utilization. The argument was first made by Amadeo (1986) and Ciccone (1986), and was picked up by Hein *et al.* (2012) and by Lavoie (1995, 1996b, 2010, 2014) in a number of papers, on the grounds that the normal rate is a convention adopted by the accountants of the firms that ought to change with past trends. More recently, more sophisticated arguments have been provided by Dutt (1997), Nikiforos (2016) and Tavani and Petach (2018), which have been endorsed by Dávila-Fernández *et al.* (2017). Thus, the introduction of equation [19] in a neo-Kaleckian growth model is not something new. Combining equations [18] and [19] in such a model is not new either: it was vaguely suggested as a possibility in Lavoie (1992, p.330) and formalized in Lavoie (1995, 1996b), in Cassetti (2006) and in Hein *et al.* (2012).<sup>6</sup> The novel feature of the present study is that we introduce for the first time the combined adjustment mechanisms given by equations [18] and [19] into the recent Kaleckian literature incorporating autonomous non-capacity creating expenditures.

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<sup>6</sup> The empirical support for this combination has been provided by Lavoie *et al.* (2004), where they claimed the relevance of the Kaleckian investment equation with hysteresis. Further empirical support can be found in Schoder (2012), Nikiforos (2016) and Setterfield (2017). In addition, Fiebiger (2018b) shows that there is a tight visual relationship between the HP-filtered data representing industry-level rates of utilization and the growth rate of industry value added, thus suggesting some degree of endogeneity in the normal rate of utilization.

As a consequence of the introduction of [19], which makes endogenous the normal rate of capacity utilization, the long-run position of the model is not an exogenously given solution anymore, stuck at a fixed target rate of capacity utilization.

From [17] and [18], the long-run equilibrium solutions are given by:

$$g^{**} = \bar{g}_z \quad [20]$$

$$\gamma^{**} = \bar{g}_z \quad [21]$$

However, there is a caveat. Equations [18] and [19] are not orthogonal, since we have from [10]:

$$\hat{\gamma} = \frac{\gamma_u \phi_\gamma}{\phi_n} \hat{u}_n. \quad [22]$$

Let us define an initial point for  $(\gamma, u_n)$  to be  $(\gamma_0, u_{n0})$ . Then the long-run equilibrium rate of utilization,  $u^{**} = u_n^{**}$ , can be obtained from [22]:

$$u^{**} = u_n^{**} = u_{n0} \left[ 1 + \frac{\phi_n(\bar{g}_z - \gamma_0)}{\phi_\gamma \gamma_u \gamma_0} \right] \quad [23]$$

This clearly shows that the final position of the system in its long-run is affected by its initial condition. In this respect, our long-run equilibrium becomes a path-dependent equilibrium.

The long-run values of other variables can be determined. Straightforwardly from [7-1], the long-run share of profits,  $\pi^{**}$ , is equal to

$$\pi^{**} = \frac{r_n v}{u_n^{**}} \quad [7-3]$$

As to the long-run income shares of direct wages,  $d^{**}$ , and overhead salaries,  $m^{**}$ , they are:

$$d^{**} = \frac{u_n^{**} - r_n v}{u_n^{**} + \sigma f} \quad [5-1]$$

$$m^{**} = \frac{\sigma f}{u_n^{**}} \left[ \frac{u_n^{**} - r_n \nu}{u_n^{**} + \sigma f} \right] \quad [6-1]$$

Then from [2], we have

$$z^{**} = [s_f(1 - s_h) + s_h] r_n + s_h \frac{\sigma f}{\nu} \left[ \frac{u_n^{**} - r_n \nu}{u_n^{**} + \sigma f} \right] - \bar{g}_z \quad [24]$$

It is shown in Appendix 3 that, essentially, in this model of autonomously growing non-capacity creating expenditures with overhead labour costs, just as in the standard model without such costs, the presence of the short-run Keynesian stability condition, which is  $A > 0$ , also guarantees dynamic stability towards a long-run fully-adjusted position, provided there exists a Harroddian adjustment mechanism tied to the investment function and provided this Harroddian instability mechanism is sufficiently weak so that it gets tamed by the growth of autonomous expenditures.

We have seen that  $\phi_\gamma$  should be small enough to meet the stability requirement. However, as is clear from [18],  $\phi_\gamma$  also governs the adjustment speed of  $\gamma$ . The smaller is the value of  $\phi_\gamma$ , the more the system is likely to be stable, but also the slower will be its process of convergence. We believe this possible trade-off between the *probability* and the *speed* of long-run convergence needs to be carefully handled.

Finally, we evaluate some of the important partial derivatives of  $z^{**}$ :

$$\frac{\partial z^{**}}{\partial r_n} = s_f(1 - s_h) + \frac{s_h u_n^{**}}{u_n^{**} + \sigma f} > 0 \quad [25]$$

$$\frac{\partial z^{**}}{\partial \sigma} = \frac{s_h f u_n^{**} d^{**}}{\nu(u_n^{**} + \sigma f)} > 0. \quad [26]$$

Recall that the numerator of  $z$  grows at a constant rate. Hence, [25] implies that when there is an upward adjustment to the target rate of return set by firms, on average the stock of capital grows more slowly than do autonomous expenditures (which grow at the rate  $\bar{g}_z$ ) during the whole traverse between the old and the new long-run equilibrium. Considering the transition dynamics after the change, we can say that the average rate of accumulation has gone down. This is indicative of the wage-led nature of growth in the long run, in the restricted sense that the rate of capital accumulation *on average* gets

reduced during the traverse, until it reaches a new long-run position after an increase in the normal rate of profit. This means that the *level* of output will be lower than otherwise it would have been if firms had not decided to raise the target rate of return. What we have is a level effect, similar to those effects identified in empirical work trying to ascertain the demand regime of an economy.

Likewise, for the same reasons, [26] implies that a widening gap in the wage obtained by overhead workers relative to that attained by direct workers slows down the accumulation of capital during the traverse to the new long-run solution. During the transition period from the initial to the final state, the average rate of accumulation goes down after an increase in the wage differential between supervisory and non-supervisory workers. This can be interpreted as supporting the prospect for the so-called ‘income (or wage) equality-led growth’ and/or ‘solidarity wage policy’.

Last but not least, the wage-led nature of growth in the present model economy, which is evident from [25], can be shown to be more prominent when wage distribution is more equitable, and hence when the relative wage gap is smaller. This can be demonstrated by [27] below:

$$\frac{\partial}{\partial \sigma} \left[ \frac{\partial z^{**}}{\partial r_n} \right] < 0. \quad [27]$$

In words, an economy where wage distribution is highly unequal, all else equal, will turn out to be less strongly wage-led in the long run. The positive association that our paper reveals between the wage-led feature of growth and a more equal wage distribution is in line with the recent claims of Carvalho and Rezai (2016) and Palley (2017).



## 5. A NUMERICAL EXAMPLE

To illustrate the analytical results shown in the previous sections, we assume an artificial economy with its long-run characteristics laid out in Table 3. The assumptions for the employment share of overhead labour and the wage premium are consistent with the data that was presented in Table 1 in the introduction.

*Table 3: Assumptions for an artificial economy*

Rate of growth	3%
Investment-to-Output ratio	10%
Profit share	33%
Employment share of overhead labour at full capacity	20%
Wage premium	4x
Normal rate of utilization of capacity	80%

We calibrate our model economy so that we can have a long-run equilibrium which is consistent with Table 3. The capital to capacity ratio,  $v$ , is set to be 2.666 since, using the definition found below, the first term is 0.1, the second term is 0.8, and the third term is 0.03, where ‘LR’ stands for the long-run.

$$v = \left[ \frac{dK}{q} \right]_{LR} \times \left[ \frac{q}{q_{fc}} \right]_{LR} \div \left[ \frac{dK}{K} \right]_{LR}$$

Then we can set,  $r_n$ , the normal rate of profit to be 0.0990 in accordance with its definition  $r_n = \pi u_n / v$ . Also we can determine the value of parameter  $f$  to be 0.2 as follows:

$$f = \left[ \frac{q/L_v}{q_{fc}/L_f} \right]_{LR} = u_{LR} \frac{L_f}{L_v} = 0.8 \times \frac{0.2}{0.8}$$

As to the value of parameter  $\sigma$ , we assume it to be 4, by considering that the overhead salary rate is close to the quadruple of the wage rate obtained by direct workers. If we normalize the productivity of direct labour to be one, i.e.,  $y_v = 1$ , then we have  $y_f = 5$ . Straightforwardly from [5-1] and [6-1], we have:

$$d = m = 0.335.$$

The equality of those two income shares is roughly consistent with Table 1. As can be seen in [11], the choice of  $(s_f, s_h)$  matters for the stability of the model. After some trials and errors, we fix  $s_f = 0.4$  and  $s_h = 0.1$ . However, we find that the main message that we report in this paper does not critically depend on this choice of  $(s_f, s_h)$ . Our purpose is to illustrate the dynamics of the main cases that we have previously identified.

The parameters that govern some of the aspects of the long-run dynamics of the model are  $(\phi_\gamma, \phi_n)$ . It should be noted that (i)  $\phi_\gamma$  needs to be small enough in order to have a stable long-run equilibrium, and that (ii) the values of  $\phi_\gamma$  and  $\phi_n$  themselves affect the endogenous normal rate of utilization of capacity,  $u_n$ , as well as (iii) the speed of convergence. Here, we apply different values for  $\phi_\gamma$  to compare the faster convergence case with relatively high values for  $\phi_\gamma$  and the slower convergence case with relatively low values for  $\phi_\gamma$ , while fixing  $\phi_n$  to be 0.2.

The initial values for  $\bar{g}_z$  and  $z$  are set to be 0.03, based on Table 3 and [24], respectively. Furthermore we set  $\gamma_u = 0.08$ , and we assume the following starting values  $\gamma = 0.03$  and  $u_n = 0.8$  for the short-run.

Table 4 shows illustrative changes in the short-run equilibrium after a permanent change in either  $r_n$  or  $\sigma$ , all else being equal. We assume that our model economy starts from an initial long-run equilibrium.

We see that a decrease in the profit rate targeted by firms, which we can suppose to be affected by the relative bargaining power of capital and labour, has a short-run expansionary effect on our calibrated model economy. This result, which we have already demonstrated analytically in [12], corresponds to the wage-led nature of the model. This is reflected in the corrected share of profits,  $\pi_c$ , which is negatively correlated with  $u$ , as it should be. By contrast, the uncorrected share of profits,  $\pi$ , is positively correlated with the rate of utilization,  $u$ .<sup>7</sup> But this should not be interpreted as proving that our model

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<sup>7</sup> Thus, in this case, a decrease in the target rate or return has led to an increase in the profit share. This possible paradoxical result arising from the presence of overhead labour costs was noted by Rowthorn (1981) and Lavoie (1995).

economy is in a profit-led regime, thus showing on this basis that empirical studies are likely to be biased towards finding profit-led demand regimes.

*Table 4: Short-run comparative statics*

	$d$	$m$	$\pi$	$\pi_c$	$u$	$g$
<b>Initial state</b>	0.3350	0.3350	0.3300	0.3300	0.8000	0.0300
Change in $r_n$ from 0.0990 to 0.0970 (2% decrease in the normal rate of profit)						
<b>After the change</b>	0.3383	0.3285	0.3332	0.3234	0.8238	0.0319
Change in $\sigma$ from 4.0000 to 3.6000 (10% decrease in the wage gap)						
<b>After the change</b>	0.3526	0.3109	0.3365	0.3300	0.8167	0.0313

In addition, we see that a more equitable wage distribution has a short-run expansionary effect. A decrease in the wage gap between overhead and direct labour yields increases in both the rate of utilization of capacity and the rate of accumulation of capital. This is indeed obvious from [14-2] since our model economy is assumed to be initially at its long-run position where:

$$u_n > \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n.$$

Unless the economy is quite far away from its long-run position (meaning that the initial rate of utilization is much below its normal rate), it seems that a widening wage gap will exert a shrinking effect on economic activity in the short run.

Table 5 is an illustration of the long-run comparative statics. We compare the initial and final long-run positions after a permanent change in either  $r_n$ ,  $\sigma$ , or  $\bar{g}_z$ , holding other parameters constant. Here, the column for  $\pi_c$  is removed since it coincides with  $\pi$  in the long run, as seen in [8-1] and [7-3].

Table 5: Long-run comparative statics

	$d$	$m$	$\pi$	$u$	$g$	$z$
<b>Initial state</b>	0.3350	0.3350	0.3300	0.8000	0.0300	0.0256
Change in $r_n$ from 0.0990 to 0.0970 (2% decrease in the normal rate of profit)						
<b>After the change</b>	0.3383	0.3383	0.3234	0.8000	0.0300	0.0248
Change in $\sigma$ from 4.0000 to 3.6000 (10% decrease in the wage gap)						
<b>After the change</b>	0.3526	0.3174	0.3300	0.8000	0.0300	0.0251
Change in $\bar{g}_z$ from 0.0300 to 0.0303 (1% increase in the growth rate of autonomous demand) with faster convergence ( $\phi_\gamma = 0.9$ )						
<b>After the change</b>	0.3441	0.3348	0.3211	0.8222	0.0303	0.0256
Change in $\bar{g}_z$ from 0.0300 to 0.0303 (1% increase in the growth rate of autonomous demand) with slower convergence ( $\phi_\gamma = 0.7$ )						
<b>After the change</b>	0.3467	0.3347	0.3186	0.8286	0.0303	0.0256

When we compare only the initial and the final long-run positions, neither the rate of utilization nor the rate of accumulation is affected by any change in  $r_n$  or  $\sigma$ .<sup>8</sup> However, this change brings about a change in  $z$ , being indicative of some important level effects on the capital stock and on total output, along with a change in the distribution of income.

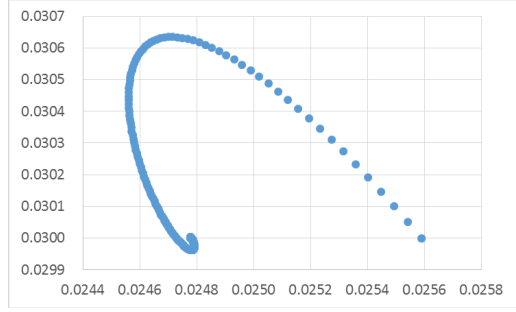
More interestingly, a permanent change in the growth rate of autonomous demand is shown to affect the long-run position in accordance with [23]. In this case, increases in the rate of utilization and accumulation are accompanied with decreases in the share of profits, suggesting again the wage-led nature of long-run growth in our model.

Now let us move our attention to the whole transition process in our calibrated economy after a change in either  $r_n$ ,  $\sigma$ , or  $\bar{g}_z$ , until it converges to a new fully-adjusted long-run position. Figure 4A and Figure 4B exhibit the transitional dynamics after a decrease in the normal rate of profit. When firms permanently lower their target profit rate, we see that the rate of utilization stays at a higher level than its initial normal rate for a while and then gradually comes back to its initial normal rate in the long run.

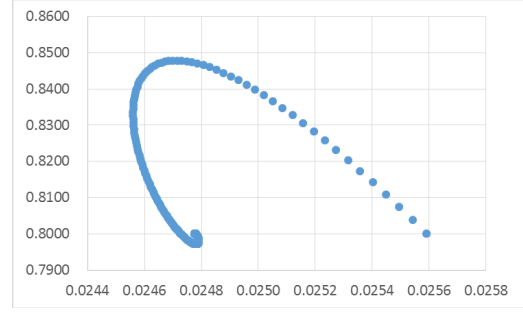
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<sup>8</sup> This is because we assumed as starting values  $\gamma = \bar{g}_z$ . Hence, as shown in [23], the long-run normal rate of utilization remains at its initial value.

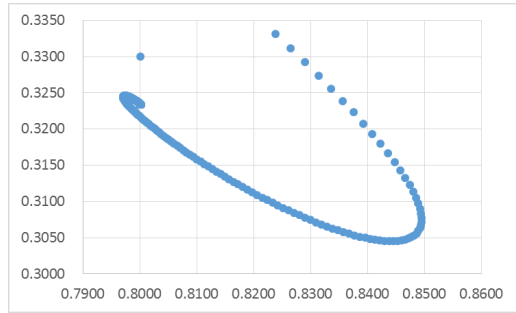
Figure 4A: Transitional dynamics after a change in  $r_n$  from 0.099 to 0.097 with  $\phi_\gamma = 0.9$



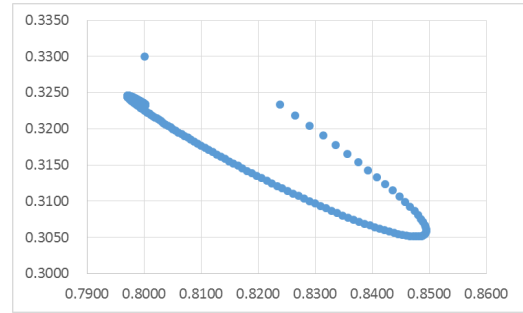
(a)  $\gamma$  (vertical) and  $z$



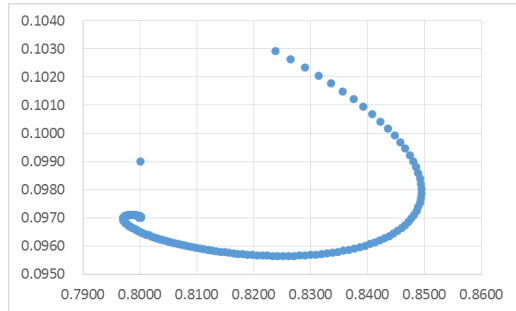
(b)  $u_n$  (vertical) and  $z$



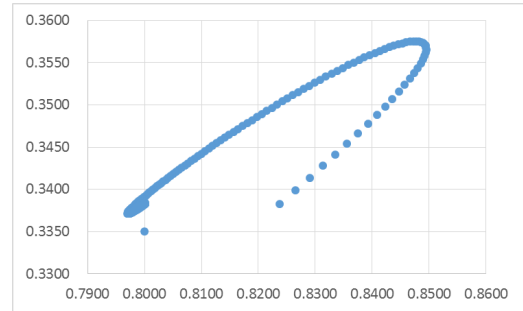
(c)  $\pi$  (vertical) and  $u$



(d)  $\pi_c$  (vertical) and  $u$

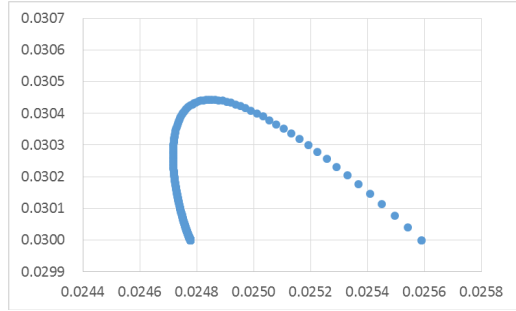


(e)  $r$  (vertical) and  $u$

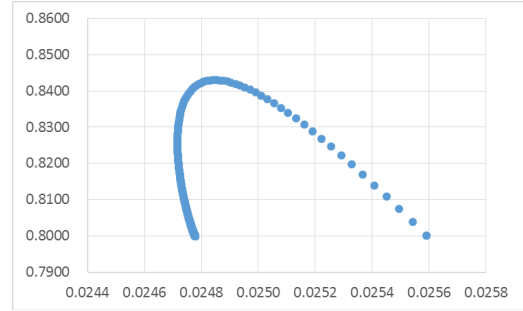


(f)  $d$  (vertical) and  $u$

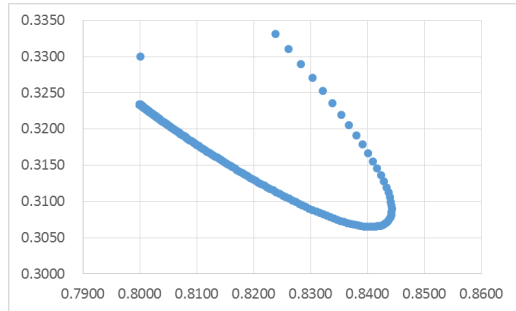
Figure 4B: Transitional dynamics after a change in  $r_n$  from 0.099 to 0.097 with  $\phi_\gamma = 0.7$



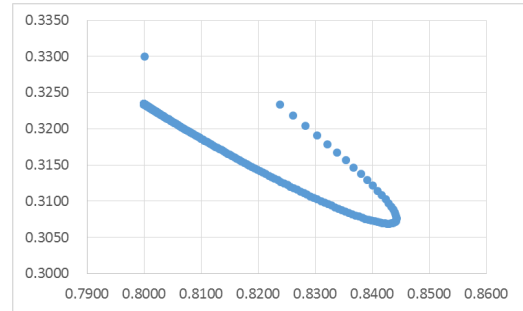
(a)  $\gamma$  (vertical) and  $z$



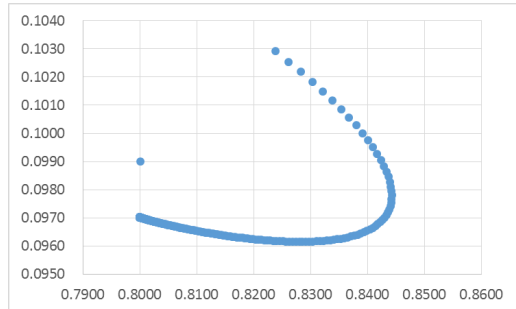
(b)  $u_n$  (vertical) and  $z$



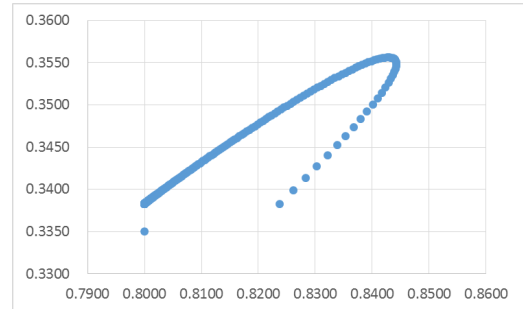
(c)  $\pi$  (vertical) and  $u$



(d)  $\pi_c$  (vertical) and  $u$



(e)  $r$  (vertical) and  $u$

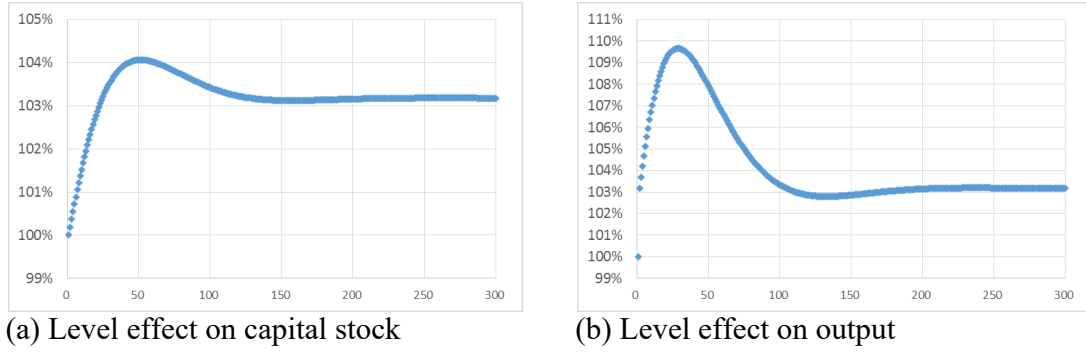


(f)  $d$  (vertical) and  $u$

One thing that stands out is that we generate here a clockwise loop in  $(u, \pi)$  space. Also, it seems that whether the share of profits is corrected or not matters in accounting for the very early phase of expansion. The uncorrected one,  $\pi$ , increases initially, and this at first glance seems to indicate that an increase in  $\pi$  contributes to an initial increase in the rate of profit together with an increase in the rate of utilization. However, the corrected one,  $\pi_c$ , does not go up when the economy expands as we can see in (d). We know that, if  $\pi$  increases while  $\pi_c$  does not, then this increase in  $\pi$  is solely due to an increase in the rate of utilization. Thus, comparing (c) and (d), we can conclude that the increase in the rate of profit during the initial expansionary phase reflects the increase in

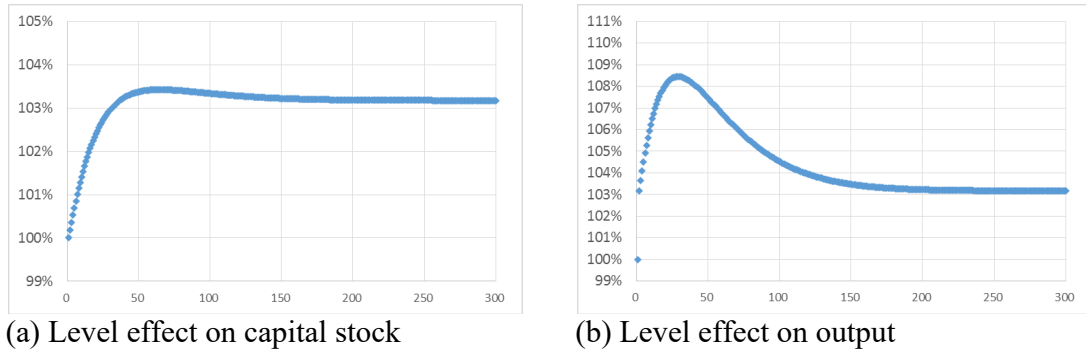
effective demand caused by the decrease in the target rate of return of firms, which could arise as a consequence of the strengthening bargaining power of labour unions or as the consequence of foreign competition.

*Figure 4C: Long-run level effect of a change in  $r_n$  from 0.099 to 0.097 with  $\phi_\gamma = 0.9$*



Note: 100% = the case of no change in  $r_n$

*Figure 4B: Long-run level effect of a change in  $r_n$  from 0.099 to 0.097 with  $\phi_\gamma = 0.7$*



Note: 100% = the case of no change in  $r_n$

Figure 4C and Figure 4D exhibit the long-run level effects on the capital stock and output of a permanent decrease in  $r_n$ . The graph describes the level of capital and output relative to the case of no change in  $r_n$ . For instance, when  $\phi_\gamma = 0.9$ , the long-run output level with  $r_n = 0.097$  is around 3% higher than the one with  $r_n = 0.099$  under the current calibration. Apparently, the increase in the relative bargaining position of workers, which may result in a decrease in the normal rate of profit, seems to have a long-run expansionary effect on the economy.

Figure 5A: Transitional dynamics after a change in  $\sigma$  from 4.0 to 3.6 with  $\phi_\gamma = 0.9$

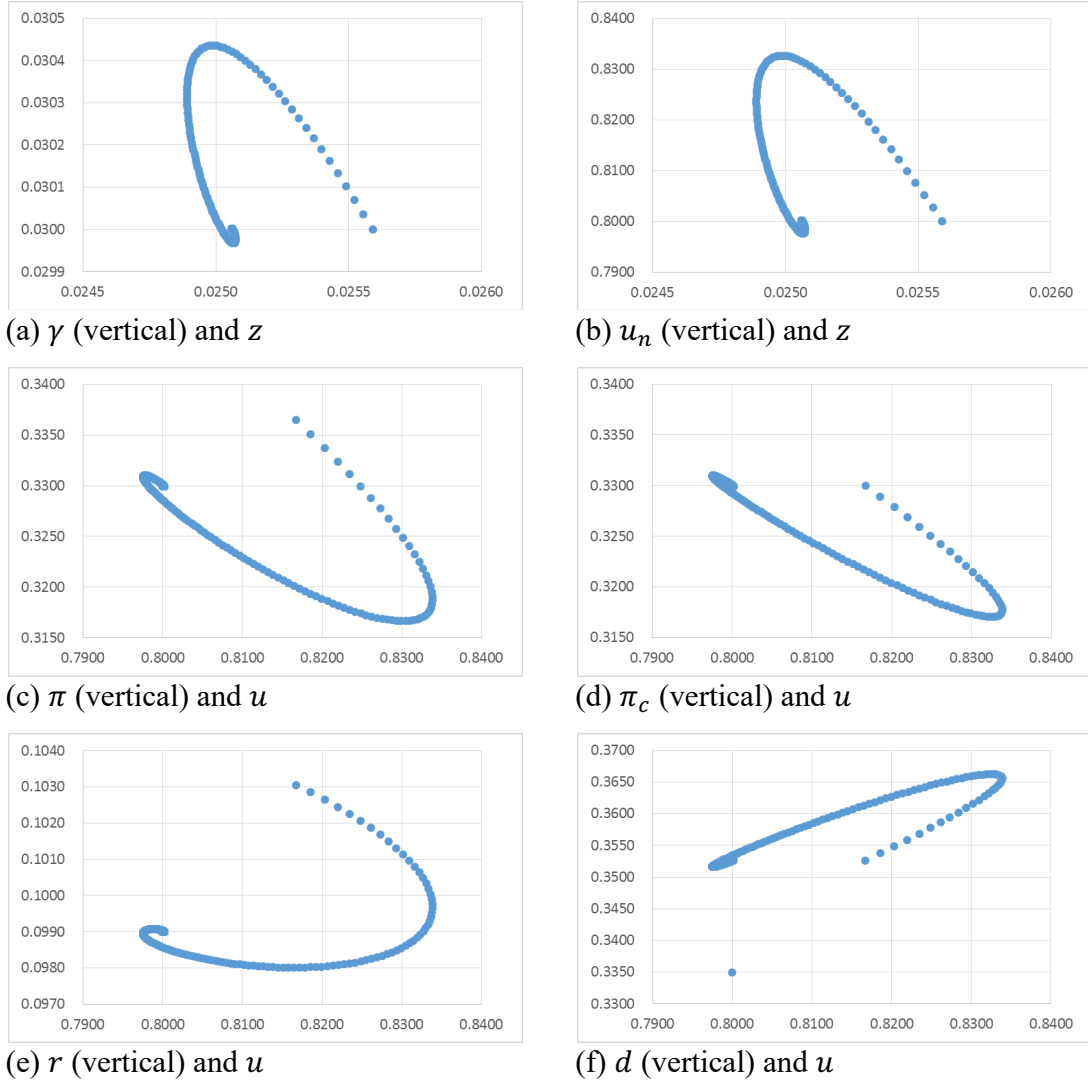
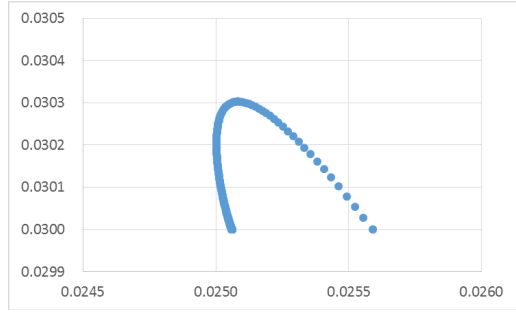


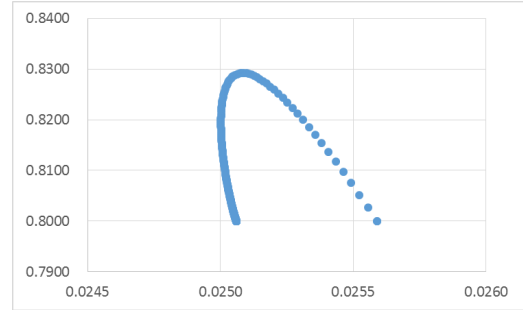
Figure 5A and Figure 5B show the transitional dynamics after a decrease in the wage premium. We can see that the rate of utilization initially increases and then comes back to its initial level in the long run. Once more, despite the wage-led nature of our model economy, the change generates a clockwise loop in  $(u, \pi)$  space. The initial increase in the rate of profit in the early expansion is entirely due to the increase in the rate of utilization. Figure 5C and Figure 5D also illustrate the positive level effects of a more equitable wage distribution on the capital stock and on output.



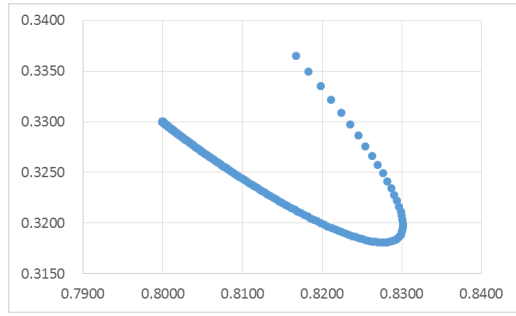
Figure 5B: Transitional dynamics after a change in  $\sigma$  from 4.0 to 3.6 with  $\phi_\gamma = 0.7$



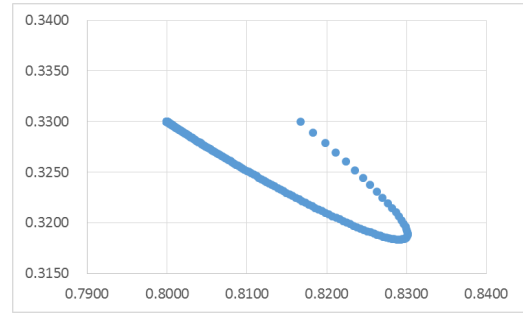
(a)  $\gamma$  (vertical) and  $z$



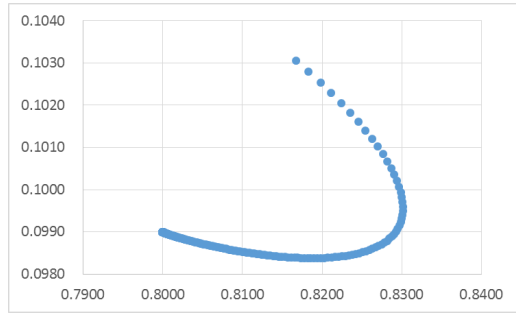
(b)  $u_n$  (vertical) and  $z$



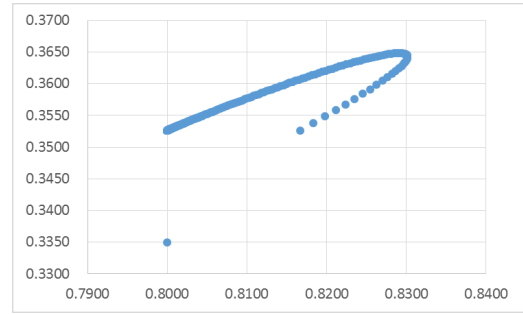
(c)  $\pi$  (vertical) and  $u$



(d)  $\pi_c$  (vertical) and  $u$

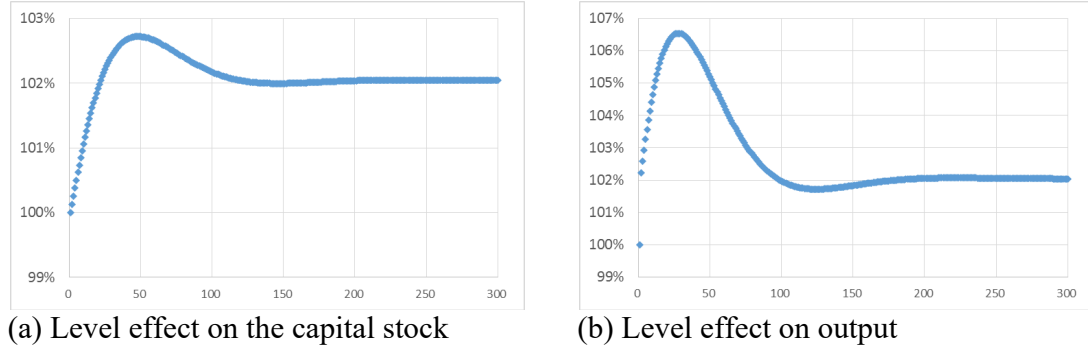


(e)  $r$  (vertical) and  $u$



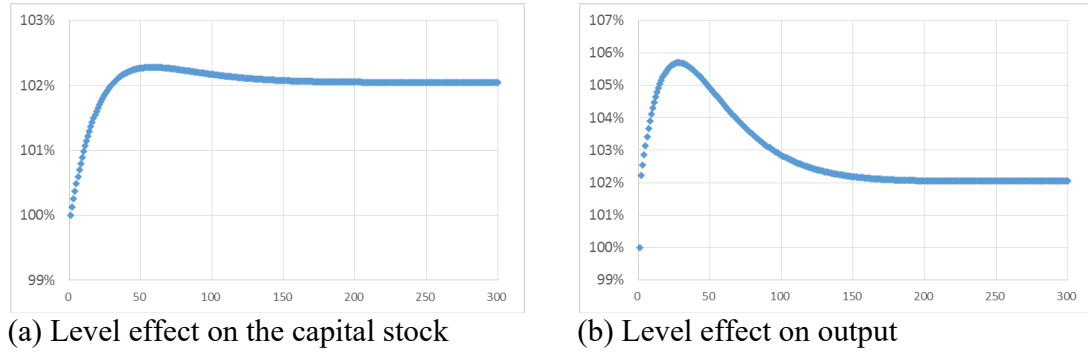
(f)  $d$  (vertical) and  $u$

Figure 5C: Long-run level effects of a change in  $\sigma$  from 4.0 to 3.6 with  $\phi_\gamma = 0.9$



Note: 100% = the case of no change in  $\sigma$

Figure 5D: Long-run level effects of a change in  $\sigma$  from 4.0 to 3.6 with  $\phi_\gamma = 0.7$

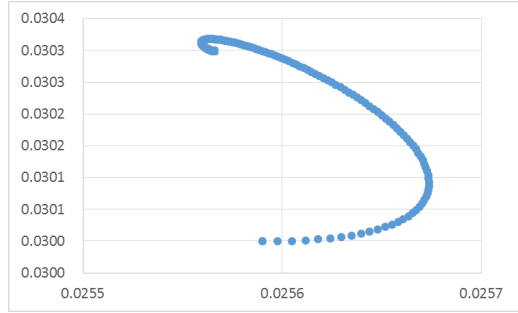


Note: 100% = the case of no change in  $\sigma$

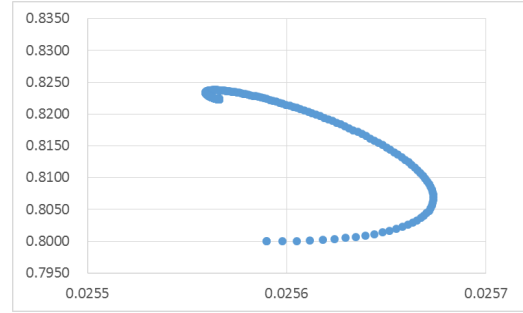
Finally, Figure 6A, Figure 6B, Figure 6C and Figure 6D illustrate the case of a permanent increase in the growth rate of autonomous consumption. Such a change generates an increase in the rate of capacity utilization and a fall in the profit share. It is particularly interesting to note that the faster growth rate of autonomous consumption generates a dynamic evolution of the profit rate relative to the rate of utilization that takes a bell shape when the profit rate is measured on the vertical axis. This dynamic bell-shaped relationship is remindful of the shape of the so-called *distributive curve* assumed by a variety of authors, who argued, on the basis of observed behaviour, that there would be a positive relationship between the profit rate and the rate of utilization at low rates of utilization, and a negative relationship at high rates of utilization, thus claiming that the economy would be wage led at low rates of utilization and profit led at high rates (Gordon 1994, p.152; You 1994, p.222; Nikiforos and Foley 2012). Here the profit rate is

fully endogenous and its bell-shaped evolution is not caused by any change in the bargaining position of the various income classes.

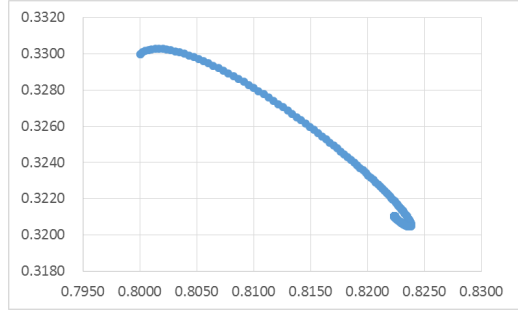
*Figure 6A: Transitional dynamics after a change in  $\bar{g}_z$  from 0.0300 to 0.0303 with  $\phi_\gamma = 0.9$*



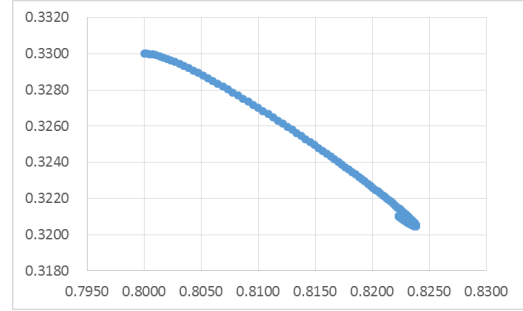
(a)  $\gamma$  (vertical) and  $z$



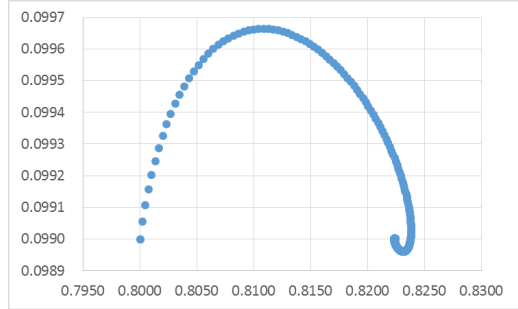
(b)  $u_n$  (vertical) and  $z$



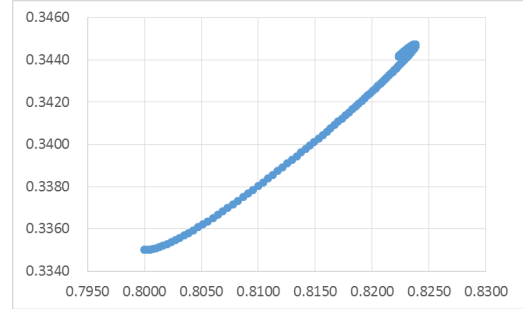
(c)  $\pi$  (vertical) and  $u$



(d)  $\pi_c$  (vertical) and  $u$

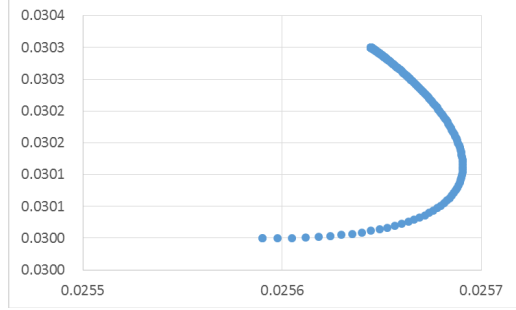


(e)  $r$  (vertical) and  $u$

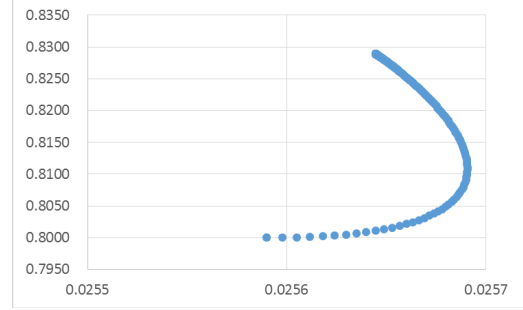


(f)  $d$  (vertical) and  $u$

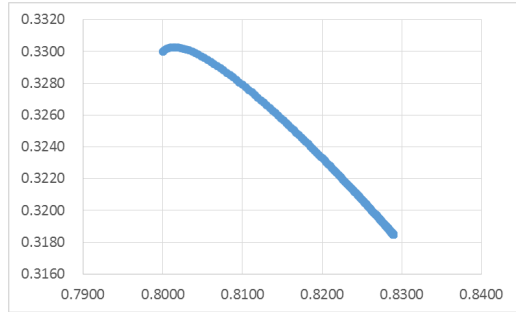
Figure 6B: Transitional dynamics after a change in  $\bar{g}_z$  from 0.0300 to 0.0303 with  $\phi_\gamma = 0.7$



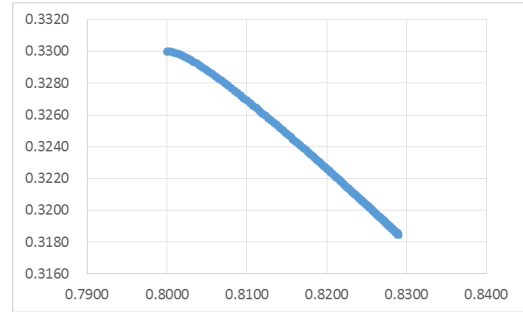
(a)  $\gamma$  (vertical) and  $z$



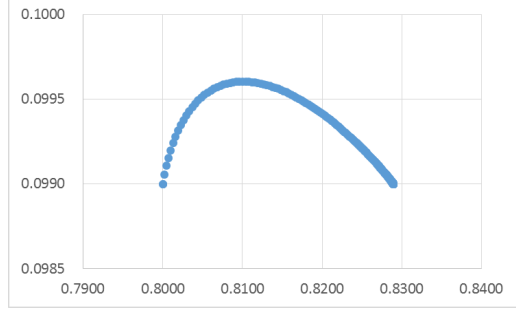
(b)  $u_n$  (vertical) and  $z$



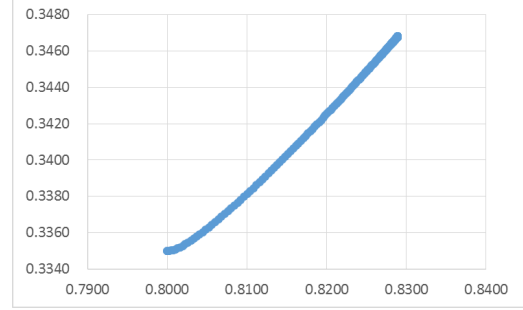
(c)  $\pi$  (vertical) and  $u$



(d)  $\pi_c$  (vertical) and  $u$

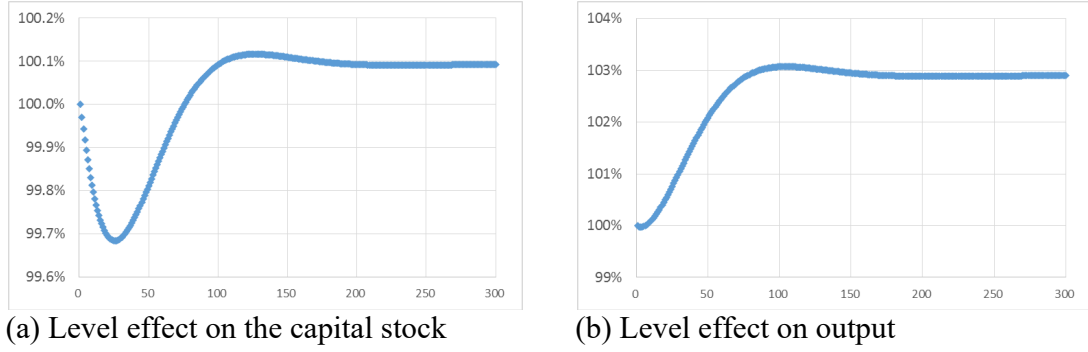


(e)  $r$  (vertical) and  $u$



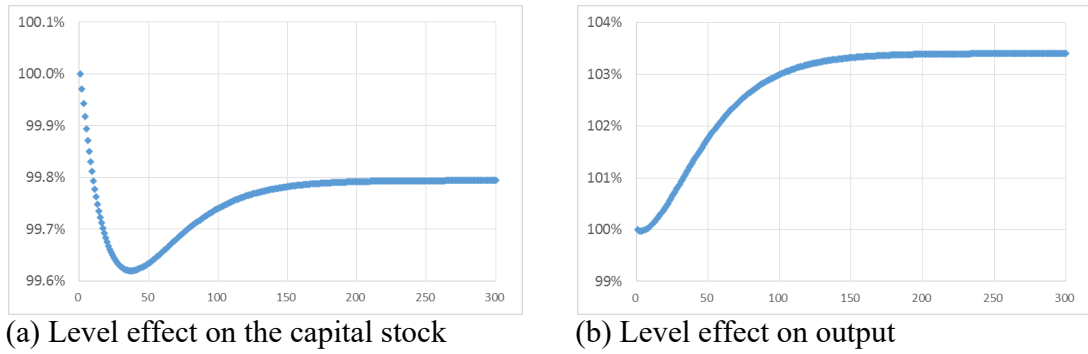
(f)  $d$  (vertical) and  $u$

Figure 6C: Long-run level effects of a change in  $\bar{g}_z$  from 0.0300 to 0.0303 with  $\phi_y = 0.9$



Note: 100% = the case of no change in  $\bar{g}_z$

Figure 6D: Long-run level effects of a change in  $\bar{g}_z$  from 0.0300 to 0.0303 with  $\phi_y = 0.7$



Note: 100% = the case of no change in  $\bar{g}_z$

## 6. CONCLUSION

We have devised a neo-Kaleckian model of growth and distribution where capital accumulation depends on the expected growth rate of sales and on the discrepancy between the actual and the normal rate of capacity utilization, but where the economy is driven by the growth rate of autonomous non-capacity creating expenditures, more specifically by the growth rate of autonomous consumption expenditures, those that Fiebigler (2018a) calls semi-autonomous household expenditures.

There are three novel features in the present model. First, we distinguish between managerial labour – white-collar or supervisory workers, the working rich – on the one hand, and direct labour – blue-collar or non-supervisory workers, the working poor – on the other hand. Blue-collar workers do not save, whereas white-collar workers have a given marginal propensity to save out of their salaries and out of the capital income they receive from firms. The white-collar workers are the social class that has autonomous consumption expenditures. The saving function is thus more complex than usual in this kind of model.

The employment of direct labour moves proportionately with the level of actual output whereas the employment of managers is tied to the level of full-capacity output. In accordance with the stylized facts noted in the introduction, there is a substantial differential between the wage rate received by managers compared to that obtained by non-supervisory workers. It follows that the profit share becomes an endogenous variable, which is a positive function of the level of capacity utilization, as is often observed in the early phase of the business cycle.

The second innovative feature of our model is that we suppose that firms set prices on the basis of a target-return pricing formula. As a consequence, an increase in the wage rate of supervisory workers relative to that of non-supervisory workers will lead to an increase in the mark-up set by firms, thus modifying in general the income distribution between profits and wages, as well as the distribution between wages going to managers and wages going to direct workers.

The third innovation of the present model with autonomous consumption expenditures is that in the long run we assume the combination of two adjustment mechanisms. As done in previous neo-Kaleckian models with autonomous non-capacity creating expenditures, we introduce a slow Harrodian instability mechanism, whereby firms increase the expected growth rate of sales when the actual growth rate is above the one expected by firms. The second long-run adjustment mechanism concerns the normal rate of utilization, which plays a role both in the investment function and in the pricing formula. It is assumed that the normal rate of capacity utilization is a convention, and that this normal rate of utilization will tend to slowly increase whenever the realized rates of utilization will surpass the normal rate.

Several interesting results directly arise from the target-return pricing formula combined to the distinction between supervisory and non-supervisory workers. First an increase the target rate of return or in the relative real wage of supervisory workers, at a given rate of capacity utilization, leads to a fall in the income share of non-supervisory workers. Second, at a given rate of capacity utilization, this increase in the relative real wage of supervisory workers will lead to an increase (decrease) in the profit share if this given actual rate of capacity utilization is bigger (smaller) than the normal rate.

Third, in an economy with overhead labour, all else equal, that is, with no change whatsoever in the mark-up over unit direct labour costs, an increase in the rate of utilization leads to a fall in the share of managerial income and an increase in the share of profits. Thus, unless the measures of the profit share are corrected for this effect, statistical enquiries will be biased towards finding that the aggregate demand regime is profit-led. We have computed an amended version of the corrected profit share suggested by Weisskopf (1979), where the share of managers is corrected by the actual rate of utilization relative to the normal rate, and have shown that this measure is both impervious to a change in the actual rate of capacity utilization and in the wage differential between supervisory and non-supervisory workers, as it ought to be when firms use a target-return pricing formula.

Combining now the above supply-side and demand-side effects, we have shown that in the short run an increase in the bargaining power of firms as reflected by their target rate of return always leads to a fall in the equilibrium rate of capacity utilization and in the rate of accumulation. The model is thus wage-led in the short run. We have also identified the conditions under which an increase in the wage differential favouring supervisory workers will lead in the short run to a fall in the rate of capacity utilization and in the rate of accumulation. This will necessarily occur when the actual rate of utilization is higher than the normal rate of utilization, but it may also happen if the actual rate of utilization is smaller than the normal rate of utilization, provided the actual rate is greater than a certain fraction of the normal rate. This fraction depends on the retention ratio of firms and on the marginal propensity to save of rich households. The higher is this marginal propensity to save of supervisory workers, the smaller will that fraction be, and hence the more likely it is that more inequality in the distribution of wages will

generate lower rates of utilization and of accumulation. This result is intuitive because the redistribution of income from non-supervisory workers, who do not save, towards supervisory workers, who do save, will reduce aggregate demand, so this negative effect will be particularly strong if the marginal propensity to save of supervisory workers is high.

As to the effect of a redistribution of income towards overhead labour on the short-run equilibrium profit share, we have shown that its effect is particularly uncertain. The only thing that can be said with certainty is that if the actual rate of utilization is in-between the same fraction of the normal rate of utilization evoked above and the normal rate of utilization, then more wage inequality will generate a fall in the short-run equilibrium value of the profit share.

Finally we consider the results obtained for the long-run equilibrium of the model. The two adjustment mechanisms involving the expected growth rate of sales (the Harroddian instability mechanism) and the normal rate of capacity utilization, combined with the dynamic behaviour of the relative weight of the autonomous consumption expenditures, drive the economy towards a fully-adjusted state, at a normal rate of capacity utilization, provided the Keynesian short-run stability condition holds and provided Harroddian instability is not overly strong. The system of dynamic equations is however path dependent so that the long-run solution depends on the initial conditions of the model as well as on the strength of the adjustment mechanisms during the transition – an interesting case of hysteresis. This is what Setterfield (1993) calls a case of *deep endogeneity*.

We have shown that an increase in the target rate of return set by firms leads to a fall in the average growth rate during the traverse to the new long-run equilibrium – a result which is consistent with what we had observed in the short run. The economy is thus clearly wage-led, a result which is not really surprising given the formulation of the investment function, given by equation [10], which does not include a term ascertaining expected profitability (at the normal rate of capacity utilization), in contrast to the post-Kaleckian model of Bhaduri and Marglin (1990).



What is more surprising and particularly interesting in the study of the long-run effects of our model is that, despite the ambiguous short-run effect on the rate of capacity utilization and the rate of accumulation of an increase in the wage differential favouring supervisory workers, there is no ambiguity with respect to the long-run effect. We have demonstrated that increased inequality in wage distribution necessarily leads to a fall in the average rate of growth of the economy during the traverse from the old to the new long-run equilibrium, thus implying a lower level of economic activity and lower potential output in the long run compared to a situation with no change in wage dispersion.

All of the above analytical results have been verified with simulations on a calibrated version of our algebraic model. In particular, the cyclical behaviour of our model has been shown in both cases of slow and fast adjustments. Despite the wage-led nature of our model economy, an increase in the target profit rate or in the share of wages going to supervisory workers generates a clockwise loop in  $(u, \pi)$  space, a kind of loop which is usually associated with a Goodwin model, with its profit-led demand regime combined to a profit-squeeze distributive curve. This is reminiscent of the apparent Goodwin cycles generated by Minskyan debt effects, as shown by Stockhammer and Michell (2017).

The main implication of these results, if the model is sufficiently faithful to reality, is that governments should pursue policies that tend to reduce wage dispersion between managerial jobs and non-supervisory jobs, as Palley (2017) also concludes. This calls for a return to egalitarian wage policies or solidarity wage policies. In other words, besides the possibility of a wage-led growth regime, our long-run results provide support for a growth strategy based on wage equality and the imposition of constraints on the perks and salaries obtained by executive managers, such as the imposition of a maximum ratio between executive income and the average or median wage of the other employees of a firm.

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### Appendix 1. Characterization of the short-run equilibrium

Taking an explicit account of our target-return pricing formula, we can rewrite our saving function as equation [2A], by making use of [6] and [7] into [2] and rearranging.

$$g_s = \frac{[s_f(1 - s_h) + s_h](\sigma f + r_n v)u - s_f(1 - s_h)\sigma f(u_n - r_n v)}{v(u_n + \sigma f)} - z \quad [2A]$$

The equilibrium rate of utilization,  $u^*$ , is then solved for from [2A] and [10], given  $(z, \gamma, u_n)$ .

$$u^* = \frac{\gamma - \gamma_u u_n + z + \frac{s_f(1 - s_h)\sigma f(u_n - r_n v)}{v(u_n + \sigma f)}}{\frac{[s_f(1 - s_h) + s_h](\sigma f + r_n v)}{v(u_n + \sigma f)} - \gamma_u}$$

Applying  $u^*$ , we can solve for the short-run equilibrium values of the share of profits,  $\pi^*$ , the rate of profit,  $r^*$ , and the rate of accumulation,  $g^*$  as follows.

$$\pi^* = \frac{(\sigma f + r_n v)u^* - (u_n - r_n v)\sigma f}{u^*(u_n + \sigma f)}$$

$$r^* = \frac{(\sigma f + r_n v)u^* - (u_n - r_n v)\sigma f}{v(u_n + \sigma f)}$$

$$g^* = \gamma + \gamma_u(u^* - u_n)$$

Note that for  $\pi^*$  and  $r^*$  to take on positive values, for otherwise unit direct and overhead costs will not be covered by the value of sales,  $u^*$  must be such that:

$$u^* > \left[ \frac{u_n - r_n v}{\sigma f + r_n v} \right] \sigma f.$$

## Appendix 2. The effect of an increase in the wage premium going to overhead labour on the short-run equilibrium profit share

Starting from equation [7], we can write

$$\frac{\partial \pi^*}{\partial \sigma} = \frac{u_n - r_n v}{u^{*2}(u_n + \sigma f)^2} \left[ \sigma f \frac{\partial u^*}{\partial \sigma} + u^*(u^* - u_n) \right].$$

This indicates that, if there is an increase in  $\sigma$ , the short-run equilibrium share of profits may either increase or get reduced, depending on the sign of the sum of the terms found inside the squared brackets on the far right. It can easily be shown that  $\partial \pi^* / \partial \sigma < 0$  if

$$\left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n \leq u^* \leq u_n.$$

This makes sense because we have already seen that, for a given rate of utilization which is lower than its normal level, an increase in the wage differential will generate a decrease in the profit share, while we have just found that if the left-hand side inequality in the relation shown above is verified, then the actual rate of utilization will fall, which means a further fall in the profit share. Otherwise, the sign of  $\partial \pi^* / \partial \sigma$  is indeterminate. The first term in the bracket is positive, while the second term is negative, if

$$u^* < \left[ \frac{s_f(1 - s_h)}{s_f(1 - s_h) + s_h} \right] u_n$$

Contrarily, the first term is negative, while the second is positive, if this time

$$u^* > u_n.$$

This explains the determination (or the indetermination) of the signs of  $d\pi$  in Table 2.

### Appendix 3. Long-run stability

Equation [24] in the text implies another inequality constraint for autonomous expenditures  $Z$  to be positive in the long run:

$$\bar{g}_z < [s_f(1 - s_h) + s_h] r_n + s_h \frac{\sigma f}{\nu} \left[ \frac{u_n^{**} - r_n \nu}{u_n^{**} + \sigma f} \right]$$

We can also see that the long-run share of saving from firms out of total saving is given by the following relation:

$$\Omega^{**} = \frac{s_f \pi^{**} u_n^{**} / \nu}{(\pi^{**} + s_h m^{**}) u_n^{**} / \nu - z^{**}}$$

This is because both the firms and the rich households should save equal proportions of their own capital stocks in the steady state, so that  $\Omega$  converges to a certain constant,  $\Omega^{**}$ .

$$\frac{s_f \pi q}{\Omega K} = \frac{s_h [mq + (1 - s_f) \pi q] - Z}{(1 - \Omega) K}$$

It is well known that, with the type of linear dependence found in [22], the Jacobian matrix,  $J$ , has a zero characteristic root, if we form  $J$  as follows:

$$J = \begin{bmatrix} \frac{\partial \hat{z}}{\partial z} & \frac{\partial \hat{z}}{\partial \gamma} & \frac{\partial \hat{z}}{\partial u_n} \\ \frac{\partial \hat{\gamma}}{\partial z} & \frac{\partial \hat{\gamma}}{\partial \gamma} & \frac{\partial \hat{\gamma}}{\partial u_n} \\ \frac{\partial \hat{u}_n}{\partial z} & \frac{\partial \hat{u}_n}{\partial \gamma} & \frac{\partial \hat{u}_n}{\partial u_n} \end{bmatrix}$$

Then, the determinant of  $J$  degenerates to zero, since it is simply the product of all the characteristic roots. With zero-root, the long-run rest point of the system is not uniquely determined. But, if we instead pin down an initial point for  $\gamma$  and  $u_n$ , and form a new Jacobian consisting only of independent components, then we can possibly have a unique long-run point and evaluate its stability with ease. Let us form a new Jacobian  $J_A$  to be:



$$J_A = \begin{bmatrix} \frac{\partial \hat{z}}{\partial z} & \frac{\partial \hat{z}}{\partial \gamma} \\ \frac{\partial \hat{\gamma}}{\partial z} & \frac{\partial \hat{\gamma}}{\partial \gamma} \end{bmatrix}$$

Now, we can evaluate each element of  $J_A$  using [17] and [18].

$$J_A = \begin{bmatrix} -\frac{\gamma_u}{B} & -1 - \frac{\gamma_u}{B} \\ \frac{\phi_\gamma \gamma_u}{B} & \frac{\phi_\gamma \gamma_u}{B} \end{bmatrix},$$

where

$$B \equiv \frac{v(u_n + \sigma f)}{A}$$

from which it follows that

$$\text{Det}(J_A) = \phi_\gamma \gamma_u v(u_n + \sigma f) A > 0$$

$$\text{Tr}(J_A) = \frac{(\phi_\gamma - 1) \gamma_u}{B}.$$

It is easy to see that the trace of  $J_A$  can be negative, so that our system turns out to be dynamically stable in its long run, under the condition that

$$\phi_\gamma < 1.$$

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