Autonomous government expenditure growth, deficits, debt and distribution in a neo-Kaleckian growth model

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Abstract
This paper is linked to some recent attempts at including a non-capacity creating autonomous expenditure category as the driver and determinant of growth into Kaleckian distribution and growth models. Whereas previous contributions have focussed on taming Harrodian instability, generated by the deviation of the goods market equilibrium rate of capacity utilisation from a normal or target rate of utilisation, we rather focus on the so far neglected issues of deficit, debt and distribution dynamics in such models. For this purpose we treat the growth of government expenditures on goods and services, financed by credit creation, as the exogenous growth rate driving the system. We examine the medium-run convergence of the system towards such a growth rate, analyse the related long-run debt dynamics and deal with stability and income distribution issues. Finally we touch upon the economic and, in particular, fiscal policy implications of our model results.

Key words: Government deficits and debt, public expenditure growth, Kaleckian distribution and growth model

JEL code: E11, E12, E25, E62

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* For helpful comments I would like to thank the participants in a workshop at the University of Cassino, 21 April 2016, and, in particular, Daniel Detzer and Achim Truger. For editing assistance I am indebted to Luisa Bunescu. Remaining errors are mine, of course.
1. Introduction

The Great Recession, following the Great Financial Crisis, has shown anew the need for fiscal policies and government deficit expenditures in order to stabilise the economy in deep recessions and to prevent a longer run depression. It has also marked the complete failure of New Consensus macroeconomics and economic policies, focussing exclusively on flexible labour markets in order to reduce the NAIRU in the long run, and on interest rate policies of the monetary authorities in order stabilise the economy in the short run (Clarida/Gali/Gertler 1999, Goodfriend/King 1997, Carlin/Soskice 2009, 2015). However, after a short ‘window of opportunity’ for fiscal policy stabilisation, we have seen a worldwide exit from the application of stabilising fiscal policies and the switch towards austerity policies, in particular in the Euro area, in an attempt to stop the increase and to reverse the trend of government deficits and debt. This has been combined with ever more desperate monetary policy interventions in money and financial markets in order to reduce long-term interest rates and to stimulate aggregate demand and growth, in a sense ‘pushing on a string’. In particular the deflationary stagnation in the Euro area since the crisis (Hein 2013/14), but also sluggish recoveries in other mature capitalist economies, indicate the failure of this approach, which has contributed to the recent discussion on ‘secular stagnation’ (Summers 2014, 2015, Hein 2016).

Post-Keynesian short-run macroeconomic models and the macroeconomic policy implications derived from these models over the last decades, or so, have increasingly focussed on active fiscal policies, government deficits and debt, when it comes to stabilising the economy, both in the short and in the long run (Arestis 2013, Arestis/Sawyer 2003, 2004, Fontana 2009, Hein/Stockhammer 2010, 2011, Setterfield 2007). Several of these models have relied on the application of Lerner’s (1943) notion of ‘functional finance’, which holds that governments should make use of fiscal deficits/surpluses in order to compensate for private sector financial surpluses/deficits with the aim of stabilising aggregate demand at (non-inflationary) full employment levels, irrespective of the concomitant government deficit- or debt-GDP ratios. This is also the macroeconomic core of what has become known as modern money theory, linking a chartalist view on money with the concept of functional finance, and, nowadays, the notion of the government as an ‘employer of last resort’ (Tcherneva 2009, 2014, Wray 2012).

Long-run debt and distribution dynamics have not been explored in much detail in these models and approaches. Several models have referred to the results by Domar (1944), who had shown that, with a constant rate of growth of nominal GDP, a constant government deficit-GDP ratio will lead to the convergence towards a constant government debt-GDP ratio. Furthermore, with the nominal rate of interest on government debt below nominal GDP growth, no primary surpluses and thus no tax revenues are required in order to satisfy government interest payment requirements to the rentiers, the holders of government debt.\(^1\)

Of course, stock-flow consistent models allow for the systematic treatment of government deficits and debt dynamics. Following Godley/Lavoie (2007a, Chapter 11, 2007b) and Martin (2008), Lavoie (2014, Chapter 5.6.3) has shown that the sustainability of

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\(^1\) For a rudimentary debate on government debt dynamics, based on the assumption of a stationary economy, see Palley (2015a, 2015b) and Tymoigne/Wray (2015). Sardoni (2016) points out to the strange assumptions being made in this debate and provides some solutions in a simple growth context drawing on Domar (1944).
functional finance along an exogenously given full-employment growth path and a convergence towards a constant government debt-income ratio is possible under less restrictive conditions than put forward by Domar (1944), if private consumption out of wealth is included into the model. However, neither investment of firms nor issues of functional income distribution are addressed in detail. Ryoo/Skott (2013) have dealt with public debt and full employment in a stock-flow consistent model, and they have included distribution issues; however, they start from a basic Harrodian approach with a given normal rate of capacity utilisation attained in equilibrium.

In this paper we will provide an account of government deficit and debt dynamics, as well as functional income distribution effects in the context of a neo-Kaleckian distribution and growth model. Our paper has been motivated by the recent contributions by Allain (2015) and Lavoie (2016), who have introduced the notion of an exogenous/autonomous growth rate of a non-capacity creating expenditure component into otherwise Kaleckian distribution and growth models. They have shown that, under weak conditions, in such models Harrodian instability, generated by the deviation of the goods market equilibrium rate of capacity utilisation from the normal or the firms’ target rate of utilisation, will be tamed and the economy will converge towards a normal rate of capacity utilisation. Simultaneously, the model economy will maintain the main features of the neo-Kaleckian distribution and growth model, the paradox of saving and the paradox of costs. However, a lower propensity to save and a lower profit share will have positive effects only on the traverse towards the long-run equilibrium, and thus only on the long-run growth path, while the long-run equilibrium growth rate will be determined by the autonomous growth rate of a non-capacity creating demand component. But neither Allain (2015) nor Lavoie (2016) have explored the related deficit, debt and distribution dynamics. Allain (2015) takes government expenditure growth as the driver of growth, and assumes a continuously balanced budget and thus avoids the discussion of the dynamics of government deficits or debt; Lavoie (2016) refers to autonomous consumption expenditures as determining long-run growth, but only mentions potential stability problems related to household debt without exploring them in any detail. Moreover, the lack of focus on deficit and debt dynamics seems to be true for most of the literature on the Sraffian supermultiplier emanating from the pioneering work of Serrano (1995), claiming that growth in modern capitalist economies is driven by the growth of a non-capacity creating component of aggregate demand (Cesaratto 2015, Dejuan 2005, Freitas/Serrano 2015).

In this paper we will follow Allain (2015) and consider the growth rate of government expenditures on goods and services as the non-capacity creating autonomous expenditure category driving growth in the long run. But different from Allain (2015), we will assume that government expenditures are financed by credit and/or money creation. We will not consider taxation issues at all, in order to focus on deficit and debt dynamics and the related distributional effects. We will also refrain from discussing Harrodian instability issues and assume, in line with the arguments proposed in Hein/Lavoie/van Treeck (2011, 2012), that the target or normal rate of capacity utilisation is either not precisely defined in a world

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2 See, for example, Laramie/Mair (2003), Palley (2013) or You/Dutt (1996) for the inclusion of taxation issues into a Kaleckian distribution and growth models.
dominated by Keynesian fundamental uncertainty or that there are forces at work which adjust the normal rate of utilisation to the actual rate in the medium to long run.

These assumptions will allow us to focus on the deficit, debt and distributional effects of an exogenous or autonomous growth rate of government expenditures, financed by credit or money creation. From an alternative to mainstream economic policy perspective, this growth rate can be conceived as an economic policy tool which may be geared towards providing aggregate demand growth at non-inflationary full employment and hence at potential growth, thus elaborating on an economic policy proposal contained in Hein/Truger/van Treeck (2012) and Hein/Detzer (2015), for example. The model and the modelling procedure that we will develop in the following sections are close to what can be found in You/Dutt (1996). The major difference is that we suppose the government to have control over the rate of growth of its expenditures for goods and services, which is the autonomous non-capacity creating demand component driving the system, whereas You/Dutt (1996) treat government expenditures as a fraction of the capital stock as their exogenous policy variable. The constancy of this ratio implies that the government is always fully informed about the change in the private capital stock when making its expenditure decisions. We would argue that our approach is more in line with the informational conditions prevailing in a world dominated by Keynesian fundamental uncertainty. Furthermore, by means of considering different tax rates on capital and labour, You/Dutt (1996) are concerned with the after tax functional distribution of income, whereas we will only examine the outcomes for the pre-tax functional distribution of market incomes.

The paper is structured as follows. In Section 2 we will present our basic model, and in Section 3 we will analyse the properties of the short-run equilibrium in which saving and investment will adjust through changes in the rate of capacity utilisation. Section 4 will then turn towards the medium-run equilibrium and we will show that the economy will adjust towards the autonomous growth rate of government expenditures for goods and services. In Section 5 we will then turn towards the long-run equilibrium and focus on the dynamics of government debt. Section 6 will summarise the main findings and draw some economic policy implications.

2. The basic model and the modelling procedure

Our model economy is a standard neo-Kaleckian one-good closed economy model with a private and a government sector. Production of the single good which can be used for consumption and investment purposes takes place in the private sector, in which firms use a non-depreciating capital stock and direct labour as inputs applying a given fixed-coefficient production technology. The emanating output is supplied in an imperfectly competitive goods market; firms set prices marking up unit labour costs, which are constant up to full capacity output. The mark-up is mainly determined by the degree of price competition in the goods market and the bargaining power of trade unions in the labour market (Kalecki 1954, Chapters 1-2, Hein 2014, Chapter 5.2). With these determinants given, prices are inelastic with respect to demand; changes in demand will trigger changes in output and capacity utilisation. We can thus treat the price level as a constant; nominal values are hence equal to real values in what follows.

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3 See Hein (2014, Chapter 6) for an introduction to Kaleckian distribution and growth models.
Since we intend to focus on the role of government expenditures, deficits and debt, we keep the private sector of our model economy as simple as possible. We assume that all wages (W) are spent for consumption purposes (Cw), hence workers do not save. All the profits (Π) are distributed as dividends to rentiers, hence there are no retained earnings. Furthermore, we also abstract from the consideration of debt finance of the firms’ capital stock. Although firms may obtain credit as initial finance for production and investment purposes, final finance (or funding) of investment and the capital stock only consists of equity issued by the firms and held by the rentiers.4

Adding the government sector to this primitive private sector of our model economy, we abstract from taxation and assume, following Lerner (1943) and the modern chartalists, that government expenditures can either be financed by issuing money or debt. For the sake of simplicity we shall only focus on government debt (L) held by rentiers, and we assume that the rate of interest on government debt (i) is under the control of the central bank, as a part of the government. Government debt in each period increases (dL) according to the sum of government expenditures (G) and the interest payments on the accumulated stock of debt (iL). Interest payments are received by rentiers, whose total or disposable income is thus composed of distributed profits plus interest paid by the government. This rentiers’ income is partly consumed and partly saved. Saving takes place in terms of accumulating equity issued by firms (dE) and bonds issued by the government (dL). Tables 1 and 2 present the balance sheet and the transaction flow matrices of our simple model economy.

Table 1: Balance sheet matrix

<table>
<thead>
<tr>
<th></th>
<th>Workers</th>
<th>Rentiers</th>
<th>Firms</th>
<th>Government</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td>-</td>
<td>+L</td>
<td></td>
<td>-L</td>
<td>0</td>
</tr>
<tr>
<td>Equities</td>
<td>-</td>
<td>+E</td>
<td>-E</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Capital</td>
<td>-</td>
<td>K</td>
<td></td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
<td>+L+E</td>
<td>0</td>
<td>-L</td>
<td>K = E</td>
</tr>
</tbody>
</table>

4 For the distinction between initial and final finance in a monetary circuit approach see Graziani (1994), Lavoie (2014, Chapter 4.6.4), Seccareccia (1996) and Hein (2008, Chapter 10.2)
Table 2: Transaction flow matrix

<table>
<thead>
<tr>
<th></th>
<th>Workers</th>
<th>Rentiers</th>
<th>Firms’ current</th>
<th>Firms’ capital</th>
<th>Government</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>-C_W</td>
<td>-C_R</td>
<td>+C_W+C_R</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Private Investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Government expenditures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+G</td>
<td>-G</td>
</tr>
<tr>
<td>Wages</td>
<td>+W</td>
<td></td>
<td></td>
<td></td>
<td>-W</td>
<td>0</td>
</tr>
<tr>
<td>Profits/dividends</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+Π</td>
<td>-Π</td>
</tr>
<tr>
<td>Interest</td>
<td>+iL</td>
<td></td>
<td></td>
<td></td>
<td>-iL</td>
<td>0</td>
</tr>
<tr>
<td>Change in equity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-dE</td>
<td>+dE</td>
</tr>
<tr>
<td>Change in loans</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-dL</td>
<td>+dL</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Total disposable income ($Y$) in our model economy is composed of income from production ($Y_P = W + \Pi$) and financial income ($Y_F = iL$):

(1) \[ Y = Y_P + Y_F = W + \Pi + iL. \]

The share of profit in the income generated in production:

(2) \[ h = \frac{\Pi}{Y_P}, \]

is determined by firms’ mark-up pricing. It is treated as an exogenous variable in our model. The ratio between financial income and production income is denoted by:

(3) \[ \Psi = \frac{Y_F}{Y_P} = \frac{iL}{Y_P} = \frac{iL}{Y_P} = \frac{iL}{Y_P} = \frac{i\lambda}{u}, \]

and is an endogenous variable in the model. It depends on the rate of interest, which is assumed to be exogenously given, the government debt-capital ratio ($\lambda = L/K$) and the rate of capacity utilisation ($u = Y_P/K$), which will each be endogenously determined in what follows. With these definitions, the share of wages in total income ($\omega$) is given as:

(4) \[ \omega = \frac{W}{Y} = \frac{(1 - h)Y_P}{(1 + \Psi)Y_P} = \frac{1 - h}{1 + \Psi}. \]
The share of profits in total income ($\pi$) is:

$$\pi = \frac{\Pi}{Y} = \frac{hY_p}{(1 + \Psi)Y_p} = \frac{h}{1 + \Psi},$$

and the share of financial income in total income ($\varphi$) is:

$$\varphi = \frac{Y_c}{Y} = \frac{\Psi Y_p}{(1 + \Psi)Y_p} = \frac{\Psi}{1 + \Psi}.$$

Finally, the capital income share is given by:

$$1 - \omega = \frac{\Pi + Y_b}{Y} = \frac{h + \Psi}{1 + \Psi} = \pi + \varphi.$$

Regarding private investment we assume a simple neo-Kaleckian investment function, in which the decisions to invest in the capital stock ($I$) are determined by animal spirits ($\alpha$) and by capacity utilisation. The rate of capital accumulation ($g$) is thus given as:

$$g = \frac{1}{K} = \alpha + \beta u, \quad \beta > 0.$$

Since workers do not save, private saving only consists of saving out of capital income, i.e. saving out of the rentiers’ profits and interest incomes. We assume a constant propensity to save of the rentiers’ households ($s_R$). For the saving rate ($\sigma$), normalising saving ($S$) by the capital stock, we obtain:

$$\sigma = \frac{S}{K} = \frac{s_R(hY_p + iL)}{K} = s_R(hu + i\lambda), \quad 1 > s_R > 0.$$

Government expenditures for goods and services are exogenously given and are supposed to grow at a constant rate ($\gamma = dG/G$), as proposed by Allain (2015) assuming a balanced government budget in his model. We thus obtain for the government expenditures-capital ratio ($b$), which can also be seen as the primary deficit-capital ratio, because it is financed by additional credit in our model:

$$b = \frac{G}{K} = \frac{G_0 e^{\gamma t}}{K}, \quad \gamma \geq 0.$$

Based on this simple framework, the further modelling procedure is as follow. We will start with the short run in which output adjusts to demand through changes in capacity utilisation, holding the government expenditures-capital ratio constant – and assuming somewhat unrealistically that governments are always fully informed about the change in the private
capital stock when making their expenditure decisions, as in You/Dutt (1996). We will give up this assumption when moving to the medium run, in which government expenditures are the policy instrument and grow at the constant rate γ. We will analyse the response of the system towards this autonomous growth rate of non-capacity creating demand and determine the medium-run equilibrium values for our endogenous variables. For both the short and the medium run of our model, we will assume the government debt-capital ratio to be constant. The dynamics of this ratio will then be tackled in the long run of our model, and we will determine the long-run equilibrium values for the endogenous variables of the model. For each run, which should be understood as an analytical device, we will analyse the effects of changes in exogenous variables on the respective endogenous variables in turn, the latter including the indicators for functional income distribution as introduced above.

3. The short-run equilibrium
For the short-run equilibrium we have the condition that the planned leakages from the circuit of income, i.e. saving, have to be equal to the planned injections, i.e. private investment plus government expenditures for goods and services, as well as government interest payments to the private sector:

\[ \sigma = g + b + i \lambda. \]

The goods markets will clear through variations in output and capacity utilisation. For the adjustment process to be stable, saving has to be more responsive to capacity utilisation than investment. From the equations (8) and (9) we hence get the stability condition:

\[ \frac{\partial \sigma}{\partial u} - \frac{\partial g}{\partial u} > 0 \quad \Rightarrow \quad s_k h - \beta > 0. \]

In what follows, we assume this stability condition to be fulfilled. Inserting equations (8), (9) and (10) yields the short-run equilibrium rate of capacity utilisation:

\[ u^* = \frac{\alpha + b + (1 - s_k) \lambda}{s_k h - \beta}. \]

And plugging equation (13) into equation (8) gives the short-run equilibrium rate of capital accumulation, which, with a constant rate of utilisation in equilibrium and constant technical coefficients of production, is also the equilibrium rate of growth of output and income:

\[ g^* = \frac{\alpha s_k h + \beta [b + (1 - s_k) \lambda]}{s_k h - \beta}. \]

Since the government debt-capital ratio is held constant in the short run, the equilibrium rate of capacity utilisation from equation (13) also uniquely determines the functional income shares from equation (4) – (6), for given profit shares in production and given interest rates, so that we get:
As can be seen from equations (15) – (17), with a constant profit share in production, a constant interest rate and a constant government debt-capital ratio, an increase (a fall) of the short-run equilibrium rate of capacity utilisation will raise (lower) the wage share and the profit share in total income, and it will lower (raise) the financial income share.

The comparative statics of the short-run equilibrium are summarised in Table 3. As can be easily seen from the respective equilibrium values, an increase in animal spirits raises equilibrium capacity utilisation, capital accumulation and growth, and thus as well the wage share and the profit share, but it lowers the financial income share.

Table 3: Effects of changes in exogenous variables on short-run equilibrium endogenous variables

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$u^*$</th>
<th>$g^*$</th>
<th>$\omega^*$</th>
<th>$\pi^*$</th>
<th>$\varphi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$sR$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$h$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+/-</td>
</tr>
<tr>
<td>$b$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$l$</td>
<td>+</td>
<td>+</td>
<td>+/-</td>
<td>+/-</td>
<td>+/-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>+</td>
<td>+</td>
<td>+/-</td>
<td>+/-</td>
<td>+/-</td>
</tr>
</tbody>
</table>

A higher propensity to save reduces equilibrium capacity utilisation, capital accumulation and growth, i.e. the paradox of saving is valid:

\[
\frac{\partial u^*}{\partial s_R} = -\frac{(hu + i\lambda)}{s_R h - \beta} < 0, \tag{13a}
\]

\[
\frac{\partial g^*}{\partial s_R} = -\frac{\beta (hu + i\lambda)}{s_R h - \beta} < 0. \tag{14a}
\]
Since a higher propensity to save causes a lower rate of utilisation, it thus lowers the equilibrium wage and profit shares, but raises the financial income share, as can easily be seen in equations (15) – (17).

A higher profit share has depressing effects on equilibrium capacity utilisation, capital accumulation and growth, i.e. the paradox of costs is valid and our model economy is wage led in the short run, both with respect to demand and to growth:

\[
\frac{\partial u^*}{\partial h} = \frac{-s_h u}{s_h h - \beta} < 0 ,
\]

\[
\frac{\partial g^*}{\partial h} = \frac{-\beta s_h u}{s_h h - \beta} < 0 .
\]

The effects of a higher profit share in production on functional income distribution for the economy as a whole are as follows:
First, a rise in the profit share in production has a direct negative effect on the wage share in total income, which is exacerbated by the depressing effect of a higher profit share on capacity utilisation:

\[
\frac{\partial \omega^*}{\partial h} = \frac{\partial u \Psi \omega}{\partial h} \frac{1}{1 + \Psi} - 1 < 0 .
\]

Second, the effect of a rise in the profit share in production on the overall profit share is ambiguous. The direct effect will be positive, but the indirect effect via the rate of capacity utilisation will be negative, so that the overall effect may be positive or negative, and firms/capitalists may face a kind of ‘paradox of costs’, i.e. a higher profit share in production being associated with a lower profit share in total income:

\[
\frac{\partial \pi^*}{\partial h} = \frac{\partial u \Psi \pi}{\partial h} \frac{u + 1}{1 + \Psi} .
\]

Third, the effect of a rise in the profit share in production on the financial income share is unambiguously positive via the negative effect on capacity utilisation:

\[
\frac{\partial \varphi^*}{\partial h} = \frac{\partial u \varphi}{\partial h} \frac{u}{1 + \Psi} > 0 .
\]

An increase in the government expenditure-capital ratio has positive effects on short-run equilibrium capacity utilisation, capital accumulation and growth (equations (13) and (14)). Through the increase in capacity utilisation the effects on the wage share and the profit share
(equations (15) and (16)) are positive, too, whereas the effect on the financial income share is negative (equation (17)).

A higher interest rate and a higher debt-capital ratio have short-run expansionary effects on equilibrium capacity utilisation, capital accumulation and growth, because additional income is injected into the system and partly consumed by the rentiers (equations (13) and (14)). However, the effects on functional income distribution are ambiguous. A rise in the interest rate or the government debt-capital ratio has a directly negative effect on the wage share and the profit share, however, the indirect effect via rising capacity utilisation on these two shares is positive. For the financial income share, the effects are in reverse direction: the direct effect is positive, whereas the indirect effect via a higher rate of capacity utilisation is negative:

$$
(15b) \quad \frac{\partial \omega^*}{\partial \lambda} = \frac{\omega \left( \frac{\partial u}{\partial \lambda} \Psi - 1 \right)}{1 + \Psi},
$$

$$
(16b) \quad \frac{\partial \pi^*}{\partial \lambda} = \frac{\pi \left( \frac{\partial u}{\partial \lambda} \Psi - 1 \right)}{1 + \Psi},
$$

$$
(17b) \quad \frac{\partial \varphi^*}{\partial \lambda} = \frac{\varphi \left( 1 - \frac{\partial u}{\partial \lambda} \Psi \right)}{1 + \Psi}.
$$

4. The medium-run equilibrium

Let us now turn to the medium run of the model, in which government expenditures grow at the constant rate $\gamma$, and let us first examine the effect on the government expenditures-capital ratio, which is also the government deficit-capital ratio. From equation (10) we obtain for the growth rate of this ratio:

$$
(17) \quad \hat{b} = \hat{G} - \hat{K} = \gamma - g.
$$

For the medium-run equilibrium, in which the government expenditures-capital ratio has to be constant, we need $\hat{b} = 0$, and thus, inserting the short-run equilibrium rate of capital accumulation from equation (14):

$$
(18) \quad \gamma = g^* = \frac{\alpha s_R h + \beta \left[ b + (1-s_R) \lambda \right]}{s_R h - \beta}.
$$

From this we can calculate the medium-run equilibrium values for our endogenous variables as follows:
\[
\begin{align*}
(19) & \quad b^{**} = \frac{\gamma(s_R h - \beta) - \alpha s_R h - \beta (1 - s_R) \lambda}{\beta}, \\
(20) & \quad g^{**} = \gamma, \\
(21) & \quad u^{**} = \frac{\gamma - \alpha}{\beta}.
\end{align*}
\]

As can be seen from equation (21), an economically meaningful medium-run equilibrium rate of capacity utilisation requires that \(\gamma - \alpha > 0\). This is what we will assume in what follows. The medium-run equilibrium rate of capital accumulation and growth will be equal to the rate of growth of government expenditures. This medium-run equilibrium will be stable if \(\frac{\partial \hat{b}}{\partial b} < 0\).

From equations (14) and (17) we get:

\[
\begin{align*}
(22) & \quad \hat{b} = \frac{\gamma(s_R h - \beta) - \{\alpha s_R h + \beta [b + (1 - s_R) \lambda]\}}{s_R h - \beta}, \\
\end{align*}
\]

which means that:

\[
(22a) \quad \frac{\partial \hat{b}}{\partial b} = \frac{-\beta}{s_R h - \beta} < 0.
\]

The medium-run equilibrium in equations (19) – (21) is thus stable, and the economy will adjust to the equilibrium determined by the growth rate of autonomous government expenditures. Table 4 summarises the effects of changes in the exogenous variables on the medium-run equilibrium values of the endogenous variables.

| Table 4: Effects of changes in exogenous variables on medium-run equilibrium endogenous variables |
|-----------------------------------------------|---|---|---|---|---|---|
| | \(u^{**}\) | \(g^{**}\) | \(b^{**}\) | \(\omega^{**}\) | \(\pi^{**}\) | \(\varphi^{**}\) |
| \(\alpha\) | - | 0 | - | - | - | + |
| \(s_R\) | 0 | 0 | + | 0 | 0 | 0 |
| \(h\) | 0 | 0 | + | - | + | 0 |
| \(\gamma\) | + | + | + | + | + | 0 |
| \(i\) | 0 | 0 | - | - | - | + |
| \(\lambda\) | 0 | 0 | - | - | - | + |

A rise in animal spirits has no effect on medium-run equilibrium capital accumulation and growth, which is determined by the autonomous growth rate of government expenditures, but it lowers the medium-run equilibrium government expenditures-capital ratio (equation (19)) and, thus simultaneously also equilibrium capacity utilisation (equation (21)). A higher propensity to save does not affect equilibrium utilisation, accumulation and growth, but raises
the medium-run equilibrium government expenditures-capital ratio, since we assume \( \gamma - \alpha > 0 \):

\[
(19a) \quad \frac{\partial b^*}{\partial s_R} = \frac{h}{\beta} (\gamma - \alpha) + i\lambda > 0.
\]

A higher profit share in production, again, has no effect on medium-run equilibrium utilisation, accumulation and growth, but raises the medium-run equilibrium government expenditures-capital ratio:

\[
(19b) \quad \frac{\partial b^*}{\partial \theta} = \frac{s_R (\gamma - \alpha)}{\beta} > 0.
\]

A higher rate of growth of government expenditures has positive effects on equilibrium capacity utilisation, capital accumulation, growth, and the government expenditures-capital ratio (equations (19) – (21)). And a rise in the interest rate or the government debt-capital ratio does not affect equilibrium capacity utilisation, capital accumulation and growth, but lowers the equilibrium government expenditures-capital ratio (equation (19)).

Turning to income distribution (Table 4), we can start with the observation that, since the government debt-capital ratio is held constant in the medium run, the equilibrium rate of capacity utilisation from equation (21) also uniquely determines the functional income shares for given profit shares in production and given interest rates, so that we get:

\[
(23) \quad \omega^* = \frac{1 - h}{1 + \Psi} = \frac{1 - h}{1 + \frac{i\lambda}{u}}, \quad \frac{\partial \omega^*}{\partial u^*} > 0,
\]

\[
(24) \quad \pi^* = \frac{h}{1 + \Psi} = \frac{h}{1 + \frac{i\lambda}{u}}, \quad \frac{\partial \pi^*}{\partial u^*} > 0,
\]

\[
(25) \quad \phi^* = \frac{\Psi}{1 + \Psi} = \frac{\frac{i\lambda}{u^*}}{1 + \frac{i\lambda}{u^*} \frac{u^*}{i\lambda + 1}} = \frac{1}{i\lambda + 1}, \quad \frac{\partial \phi^*}{\partial u^*} < 0.
\]

Therefore, a rise in animal spirits, because it lowers medium-run equilibrium capacity utilisation, also lowers the wage and the profit share in total income, but raises the financial income share. A change in the propensity to save out of profits has no effect on utilisation, and thus also no effect on functional income distribution. A higher profit share in production does not affect equilibrium utilisation and, therefore, it does not have any impact on the distribution between financial income and income from production. However it lowers the wage share and raises the profit share in total income. A higher rate of growth of autonomous
government expenditures has a positive effect on medium-run equilibrium capacity utilisation, and therefore it raises the wage share and the profit share, and it reduces the financial income share. And finally, an increase in the interest rate or in the government debt-capital ratio does not affect equilibrium utilisation, and therefore it reduces the medium-run equilibrium wage and profit shares, and it raises the financial income share.

5. The long-run equilibrium

Turning to the long-run equilibrium, we have to examine the dynamics of the government debt-capital ratio and the respective feedbacks on our endogenous model variables. Government debt in each period rises by the sum of government expenditures for goods and services, which is equal to the primary government deficit, and government interest payments:

\[ \Delta L = G + iL. \]

For the growth rate of government debt we thus obtain:

\[ \dot{\lambda} = \frac{\Delta L}{L} = \frac{G}{\lambda} + i. \]

The rate of change of the government debt capital ratio \((\lambda = L/K)\) is:

\[ \dot{\lambda} = \dot{L} - \dot{K} = \frac{b}{\lambda} + i - \gamma. \]

In the long-run equilibrium the endogenously determined government debt-capital ratio has to be constant, and thus we need \(\dot{\lambda} = 0\). Making use of the medium-run equilibrium government deficit-capital ratio from equation (19), we get the following long-run equilibrium values:

\[ \lambda^{**} = \frac{b}{\gamma - i} = \frac{\gamma s_h h - \beta - \alpha s_h h}{\beta (\gamma - s_h i)}, \]

\[ b^{**} = (\gamma - i)\lambda^{**} = \frac{(\gamma - i)[\gamma (s_h h - \beta) - \alpha s_h h]}{\beta (\gamma - s_h i)}, \]

\[ g^{**} = \gamma, \]

\[ u^{**} = \gamma - \frac{\alpha}{\beta}. \]

With a positive and constant rate of growth of government expenditures \((\gamma)\), and a positive medium-run equilibrium government expenditures-capital ratio \((b^{**} > 0)\) from equation (19), which implies that the numerator in equation (29) is positive, we can distinguish three constellations:
1. If $\gamma > i > s_R i$, we will obtain a positive long-run equilibrium government debt-capital ratio (equation (29)), which will also be associated with a positive long-run equilibrium government expenditures (and hence primary deficit)-capital ratio (equation (30)).

2. If $i > \gamma > s_R i$, the long-run equilibrium government debt-capital ratio will still be positive, but it will require the government expenditures-capital ratio to turn negative, and hence to a primary surplus in the long-run equilibrium, the modelling of which would mean to include government revenues (taxes) into our model.

3. If $i > s_R i > \gamma$, the long-run equilibrium government debt-capital ratio will have to be negative, and it will be associated with a positive long-run equilibrium government expenditures-capital ratio. This constellation is unfeasible in our model and hence unstable.

In what follows we will only focus on constellation 1, with both positive long-run equilibrium government expenditures- and debt-capital ratios. This long-run equilibrium will be stable, if $\frac{\partial \lambda}{\partial \lambda} < 0$. From equation (28) for the growth of the government debt-capital ratio, making use of equation (19) for the medium-run government expenditures-capital ratio, we obtain:

$$\frac{\partial \lambda}{\partial \lambda} = -\frac{(1-s_R)\lambda - b^*}{\lambda^2} = -\frac{[\gamma(s_R h - \beta) - \alpha s_R h]}{\beta \lambda^2} < 0.$$  

The long-run equilibrium in constellation 1 is thus stable, and it requires that the rate of growth of government expenditures, determining the long-run growth rate of the system, exceeds the rate of interest on government debt. This means that for this constellation in our model we arrive at the same conclusion regarding the sustainability and stability of government debt as Domar (1944).

As shown in Table 5, variations in exogenous variables have the same long-run equilibrium effects on capacity utilisation, capital accumulation and growth as for the medium-run equilibrium. Changes in the rentiers’ propensity to save, in the profit share and in the rate of interest do not affect long-run capacity utilisation, capital accumulation and growth. A rise in animal spirits is detrimental to capacity utilisation, but does not affect accumulation and growth, whereas a rise in the rate of growth of government expenditures has expansionary effects on utilisation, accumulation and growth in long-run equilibrium.

| Table 5: Effects of changes in exogenous variables on stable long-run equilibrium endogenous variables |
|---|---|---|---|---|---|---|---|
| $a$ | $s_R$ | $h$ | $\gamma$ | $i$ | $u$ | $g$ | $b$ |
| $***$ | $0$ | $0$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $\omega$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $\pi$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $\phi$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |

Focussing on government expenditures, and thus primary deficits, as well as government debt next, we find the following results for the long-run equilibrium (equations (29) and (30)).
rise in animal spirits will lower both the long-run equilibrium government expenditures- and debt-capital ratios. A higher rentiers’ propensity to save will increase both ratios:

\[
\frac{\partial \lambda^{***}}{\partial \lambda} = \frac{hu + i\lambda}{\gamma - s_k i} > 0 , \tag{29a}
\]

\[
\frac{\partial b^{***}}{\partial \lambda} = \frac{(\gamma - i)(hu + i\lambda)}{\gamma - s_k i} > 0 . \tag{30a}
\]

The same is true for a rise in the profit share in production, because we have \( \gamma - \alpha > 0 \):

\[
\frac{\partial \lambda^{***}}{\partial h} = \frac{s_k (\gamma - \alpha)}{\gamma - s_k i} > 0 , \tag{29b}
\]

\[
\frac{\partial b^{***}}{\partial h} = \frac{(\gamma - i) s_k (\gamma - \alpha)}{\gamma - s_k i} > 0 . \tag{30b}
\]

A higher interest rate on government debt will mean a higher long-run equilibrium government debt-capital ratio, but a lower long-run equilibrium government expenditures-capital ratio:

\[
\frac{\partial \lambda^{***}}{\partial i} = \frac{s_k \lambda}{\gamma - s_k i} > 0 , \tag{29c}
\]

\[
\frac{\partial b^{***}}{\partial i} = \frac{\lambda \gamma (s_k - 1)}{\gamma - s_k i} < 0 . \tag{30c}
\]

A higher rate of growth of government expenditures has the following effects on the long-run equilibrium government debt- and expenditures-capital ratios:

\[
\frac{\partial \lambda^{***}}{\partial \gamma} = \frac{s_k h}{\beta} - 1 - \lambda \frac{1}{\gamma - s_k i} , \tag{29d}
\]

\[
\frac{\partial \lambda^{***}}{\partial \gamma} > 0 , \quad \text{if} : \quad \frac{s_k h}{\beta} - 1 > \lambda . \tag{29d'}
\]

\[
\frac{\partial b^{***}}{\partial \gamma} = \frac{\left( \frac{s_k h}{\beta} - 1 \right)(\gamma - i) + i\lambda (1 - s_k)}{\gamma - s_k i} . \tag{30d}
\]
This implies that a higher rate of growth of government expenditures leads to a higher long-run equilibrium government expenditures-capital ratio for the case we are considering here, i.e. \( \gamma > i > s_R i \). However, the effect on the long-run equilibrium government debt-capital ratio is not unique and depends on the value of this ratio in the initial equilibrium. If this value falls short of the threshold in (29d'), the long-run equilibrium government debt-capital ratio will rise as well. If it exceeds the threshold in (29d'), a higher rate of growth of government expenditures will cause a lower long-run equilibrium government debt-capital ratio. In this case, we have a 'paradox of debt', i.e. an increase in primary government deficits, but a fall in the government debt-capital ratio.

Regarding functional income distribution in long-run equilibrium, we have to consider that any change in exogenous variables will affect distribution through changes in long-run equilibrium capacity utilisation and the government debt-capital ratio:

\[
\omega_{***} = \frac{1-h}{1+\Psi} = \frac{1-h}{1+\frac{i\lambda_{***}}{u_{***}}}, \quad \frac{\partial \omega_{***}}{\partial u} > 0, \quad \frac{\partial \omega_{***}}{\partial \lambda} < 0, \\
\pi_{***} = \frac{h}{1+\Psi} = \frac{h}{1+\frac{i\lambda_{***}}{u_{***}}}, \quad \frac{\partial \pi_{***}}{\partial u} > 0, \quad \frac{\partial \pi_{***}}{\partial \lambda} < 0, \\
\phi_{***} = \frac{\Psi}{1+\Psi} = \frac{\frac{i\lambda_{***}}{u_{***}}}{1+\frac{i\lambda_{***}}{u_{***}}} = \frac{1}{\frac{u_{***}}{i\lambda_{***}} + 1}, \quad \frac{\partial \phi_{***}}{\partial u} < 0, \quad \frac{\partial \phi_{***}}{\partial \lambda} > 0.
\]

A rise in animal spirits will lower the long-run equilibrium rate of capacity utilisation and also the long-run government debt-capital ratio, which have opposite effects on functional income shares. However, calculating the overall effect, we obtain:

\[
(33a) \quad \frac{\partial \omega_{***}}{\partial \alpha} = \frac{1-h}{u(1+\Psi)} \left( \frac{\partial u}{\partial \alpha} \Psi - \frac{\partial \lambda}{\partial \alpha} i \right) = \frac{\beta i(1-h)}{(1+\Psi)^2(\gamma - \alpha)^2(\gamma - s_R i)} > 0, \\
(34a) \quad \frac{\partial \pi_{***}}{\partial \alpha} = \frac{h}{u(1+\Psi)} \left( \frac{\partial u}{\partial \alpha} \Psi - \frac{\partial \lambda}{\partial \alpha} i \right) = \frac{\beta i h}{(1+\Psi)^2(\gamma - \alpha)^2(\gamma - s_R i)} > 0,
\]

Note that we only discuss the case for which \( \gamma > i > s_R i \). A rise in animal spirits will thus raise the production income shares, i.e. the wage share and the profit share, and it will therefore lower the long-run equilibrium financial income share:

---

5 It should be noticed that the short-run goods market equilibrium condition implies that \( s_R h/\beta > 1 \).
A higher propensity to save will have no effect on long-run equilibrium capacity utilisation, but it will cause a higher government debt-capital ratio. Therefore, it will raise the financial income share, and will depress the wage share and the profit share in long-run equilibrium.

A higher profit share in production has no effect on long-run equilibrium capacity utilisation and it will raise the long-run equilibrium government debt-capital ratio. Therefore, it has a uniquely depressing effect on the wage share and a raising effect on the financial income share. The impact on the profit share in total income is not clear a priori, but it depends on the relative strength of the redistribution of income in production and the effect on the long-run equilibrium government debt capital ratio. If the latter is very pronounced, the increase in the profit share in production may be associated with the fall in the profit share in total income, and we will again have a kind of ‘paradox of costs’.

A higher growth rate of government expenditures increases the long-run equilibrium rate of capacity utilisation, which has an expansionary effect on the wage and profit shares, and a contractionary effect on the financial income share. Furthermore, a higher growth rate of government expenditures lowers the long-run equilibrium government debt-capital ratio, if \( \frac{s_k h}{\beta} - 1 < \lambda \) in equation (29d), which would then also contribute to a higher wage and a higher profit share, and to a lower financial income share. In this case, therefore, a higher growth rate of government expenditures would uniquely raise the long-run equilibrium wage and profit shares in total income, and it would mean a lower long-run financial income share. However, if a higher growth rate of government expenditures raises the long-run equilibrium government debt-capital ratio, i.e. \( \frac{s_k h}{\beta} - 1 > \lambda \) in equation (29d), the overall effect on functional income distribution is indeterminate and will depend on the relative strengths of the individual channels:
lies that both, the government.

Finally, a higher interest rate will have no effect on long-run equilibrium capacity utilisation, but it will cause a higher government debt-capital ratio. Therefore, a higher rate of interest will raise the financial income ratio, and it will depress the wage and the profit share in total income in long-run equilibrium.

6. Main results and economic policy implications

Summing up, our simple model contains several interesting results and provides some important messages for economic policies in general, and for fiscal policies in particular.

A constant medium- to long-run growth rate of government expenditures financed by credit creation (or money emission) will provide a stable long-run growth rate of the system to which capital stock, output and income growth will converge. This growth rate can thus be geared towards providing stable non-inflationary full employment and thus to have the system grow at its potential rate of growth, given by labour force growth and productivity growth. In particular for the latter, however, it needs to be taken into account furthermore that productivity growth to a large extent is endogenous to capital stock and GDP growth (Kaldor’s technical progress function, Verdoorn’s Law), and thus also to the autonomous growth rate of government expenditures.6

A constant medium- to long-run growth rate of government expenditures financed by credit creation (or money emission) implies that both, the government expenditures (and thus primary deficit)-capital (or -GDP) ratio and the government debt-capital (or -GDP) ratio will converge toward definite values in the medium to long run. For this to happen, the rate of interest on government debt will have to fall short of the rate of accumulation and growth (γ > i > $r_i$). If however, this condition is not met, but the rate of growth exceeds the product of the rate of interest rate and the rentiers’ propensity to save ($i > γ > s_r i$), the convergence towards a positive long-run equilibrium government debt-capital ratio will require a negative government expenditures-capital ratio, and hence primary surpluses. Any increase in the autonomous and deficit-financed rate of growth of government expenditures means a higher medium- and long-run equilibrium government expenditures-capital ratio, but it may be

6 See Dutt (2013) and Sardoni (2016) for the discussion on the relationship between government fiscal policies and productivity growth, albeit in different models from the one presented here.
associated with a higher or lower long-run equilibrium government debt-capital ratio. In other words, our model contains the possibility of a ‘paradox of debt’. The latter is the more likely the higher the initial government debt-capital ratio will be.

Overall functional distribution of market incomes is endogenously determined through the endogeneity of both capacity utilisation and the government debt-capital ratio, with the interest rate and the wage and profit shares in production exogenously given. This means that in the long run, any change in the autonomous growth rate of government expenditures has potentially contradictory effects on functional income distribution: A higher growth rate of government expenditures means a higher rate of utilisation and thus higher wage and profit shares, and a lower financial income share. However, it may, but need not, mean a higher government debt-capital ratio, which will have depressive effects on the wage and profit shares, and raise the financial income share. In particular, if the government debt-capital ratio is high in the initial equilibrium, raising the growth rate of government expenditures may trigger a lower long-run equilibrium government debt-capital ratio and thus higher wage and profit shares, and a lower financial income share.

The changes in other (economic policy) parameters than the growth rate of government expenditures have only short-run utilisation and growth effects, but, may have long-run distribution effects. A lower propensity to save and a lower profit share boost utilisation, capital accumulation and growth in the short run, but not in the medium and long run. In other words, both changes affect the traverse towards the long-run equilibrium, and thus the growth path, but not the long-run growth rate. And they change income distribution in the long run, in favour of the share of production income and at the expense of the share of financial income. A higher rate of interest, again, has short-run expansionary effects on capacity utilisation, capital accumulation and growth, because additional income is generated for rentiers, without any medium- or long-run effects, however. In the long run, only the growth path is affected, and there are distribution effects, which are in favour of the financial income share and at the expense of the production income shares. Furthermore, raising the interest rate above the autonomous growth rate of government expenditures will require primary government surpluses to stabilise the system, and it may even render the whole system unstable if this increase is associated with a very high propensity to save out of rentiers’ income.

Of course, our model should only be seen as first step in the application of the concept of autonomous government expenditures growth as the long-run growth determinant, looking simultaneously at the related deficit, debt and distribution dynamics. However, we believe that some important insights emerge with respect to deficit and debt dynamics, as well as with respect to income distribution. It remains to be examined to what extent these insight can be sustained in more complex models, which might include taxes and thus the post-tax distribution of income distribution, more complicated investment functions, explicitly considering the issue of investment finance for example, wealth-based and debt-financed private consumption, or a foreign sector.
References


